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Complementary relationship of evaporation and the mean annual water-energy balance

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[1] By combining the complementary relationship of evaporation with the coupled long-term water-energy balance of Porporato et al. (2004) in a Budyko-type framework, one can, from atmospheric measurements alone, derive important ecosystem characteristics, such as the mean effective relative soil moisture and the maximum soil water storage, as well as predict changes in the rooting depth of vegetation as a response to climate variations.

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[2] *Gerrits et al.* [2009] and *Yang et al.* [2008] just recently gave an interesting solution to the implicit problem of $E = f(P, E_0, E)$, where E is the long-term evaporation rate at the catchment scale, E_0 is a measure of the potential evaporation rate, and P is precipitation. While the approach of *Gerrits et al.* [2009] requires three parameters (three threshold values) to calibrate, that of *Yang et al.* [2008] needs only one, c, to express E/P as $E/P = f(E_0/P, c)$. Another single-parameter solution has been suggested by *Porporato et al.* [2004] from statistical and conceptual consideration of the precipitation time series and the soil water balance, with the clear advantage that their parameter can be related to properties of the precipitation process, the soil matrix and the rooting depth of the vegetation.

[3] The long-term water-energy balance solution, $E/P = f(E_0/P, c)$, can also be obtained by the application of the complementary relationship [*Bouchet*, 1963] written now in the form

$$E = 2E_0 - E_{PM}, \qquad (1)$$

where E_0 is the long-term wet environment (potential) evaporation rate, in practice almost exclusively defined by the Priestley-Taylor equation [*Priestley and Taylor*, 1972], and E_{PM} the Penman potential evaporation rate [*Penman*, 1948]. E_0 is defined as $\alpha\Delta(\Delta + \gamma)^{-1}R_n$ where α is the Priestley-Taylor parameter, Δ is the slope of the vapor saturation curve, and R_n is the net radiation. Equation (1), when Δ is evaluated at the measured air temperature (representing nonwet conditions) is the classical advection-aridity (AA) model of *Brutsaert and Stricker* [1979]. *Szilagyi and Jozsa* [2008] and *Szilagyi et al.* [2009] recommended a way to reduce the nonwet air temperature to bring it closer to wet environment conditions and evaluated the slope of the saturated vapor pressure curve at that temperature. Such a temperature modification proved to be advantageous in semiarid conditions, thus this modification of the AA model is kept here. Note also that the definition of E_0 by the wet environment evaporation rate is well justified because the long-term water-energy balance necessarily operates at the catchment scale, plus E_0 depends predominantly on the available energy only.

[4] By dividing both sides of (2) with P, one obtains

$$E/P = (2 - E_{PM}/E_0)E_0/P = f(E_0/P, c), \qquad (2)$$

where the parameter c now becomes defined as $c = 2 - E_{PM}/E_0$. *Budyko* [1974] introduced the aridity index, Φ, as $\Phi = E_0/P$, therefore (2) transforms into

$$E/P = f[\Phi, c]. \qquad (3)$$

[5] Let us see, how alternative, parameter-parsimonious formulations of the long-term water-energy balance, i.e., equation (3), those of *Yang et al.* [2008] and *Porporato et al.* [2004] as well as other classical parameter-free ones, match observations.

[6] From the 120 watersheds of the contiguous U.S. minimally disturbed by human activity [*Wallis et al.*, 1991; *Slack and Landwehr*, 1992] that were studied by *Ramirez and Claessens* [1994], and *Hobbins et al.* [2001a, 2001b], 23 that contained at least one Solar and Meteorological Surface Observation Network (SAMSON) station (Figure 1) within their boundaries (two catchments contained two stations) were selected [*Szilagyi et al.*, 2009]. For these watersheds runoff (R) data for 1961–1990 were collected from USGS gauging stations, and precipitation (P) data from the Parameter-elevation Regressions on Independent Slopes Model (PRISM) [*Daly et al.*, 1994]. To increase the number of data points, water balance closure of $E_{wb} = \langle P \rangle - R$ was performed on a 10-year basis, thus yielding $3 \times 25 = 75$ E_{wb} values. Braces here denote spatial averaging of grid precipitation values over the watershed. A 10-year period is considered long enough for water storage changes to be negligible, but short enough to still see some variations among consecutive periods.

[7] Input variables of the AA model were calculated with daily values of air and dew point temperatures, pressure,

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Figure 1. Location of the 210 SAMSON stations as well as the 23 watersheds that contain a SAMSON station.

wind velocity and incident global radiation. The routines of WREVAP [*Morton et al.*, 1985] were used to convert global radiation into net radiation (R_n) values. Subsequently, the AA model's daily E_0 and E_{PM} rates were aggregated into monthly values before E was calculated by (1) on a monthly basis followed by taking long-term averages of the terms involved, which is the same as employing (1) with long-term (i.e., 10-year) averages, since averaging is commutative with subtraction.

[8] Figure 2 displays the water balance and AA model–derived values, while Figure 3 also contains the different theoretical curves that formulate E/P as a function of Φ. The unit slope line from the left represents a theoretical upper limit for the E/P values in humid regions where evaporation is energy limited, while the horizontal line is the upper limit in arid regions, where evaporation is water limited. As could be expected the two theoretical curves that contain a free parameter yield the best fit to the observed data, either in humid or more arid regions. From the two best fit curves, the one by *Porporato et al.* [2004] is superior, as it provides for a better fit in the $0.5 < \Phi < 1$ region, where most of the data points lie.

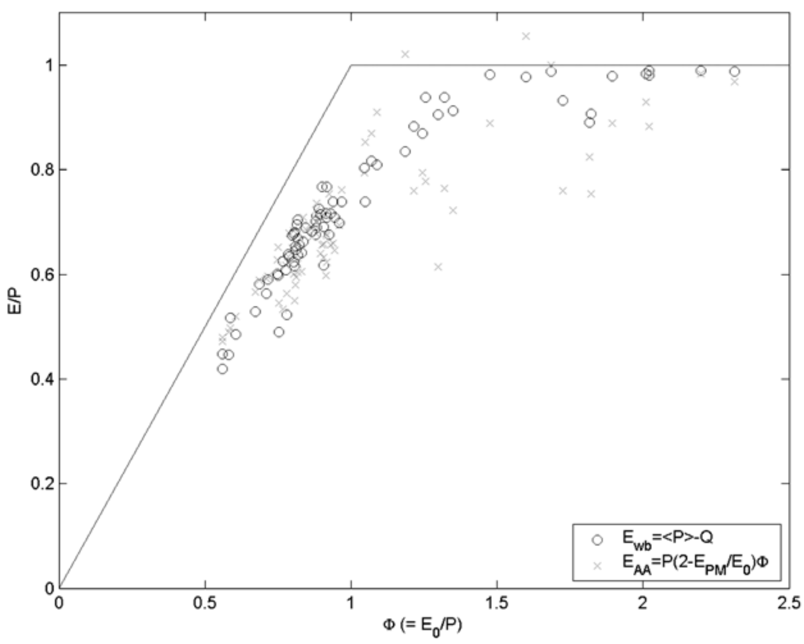

Figure 2. Ten-year average evaporation-precipitation ratios (E/P) plotted against the aridity index, Φ, for the selected watersheds, 1961–1990. E_{wb}, water balance; E_{AA}, advection-aridity model. E_0 is estimated by the Priestley-Taylor equation with $\alpha = 1.31$; E_{PM} is estimated with the Rome wind function, $f(u)$ $(0.26(1 + 0.54u_2)$, where u_2 is the mean wind speed in m s^{-1} at 2 m) [*Brutsaert*, 2005].

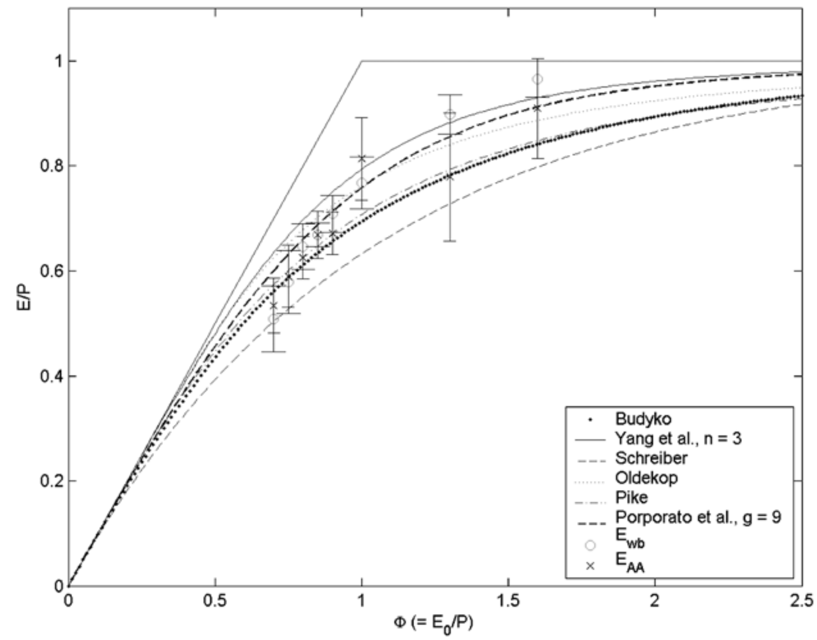

Figure 3. Whisker plot of the binned E/P values from Figure 2. The length of the whiskers equals the standard deviation of the E/P values within the bin, and the whiskers emanate up and down from the respective means. The wider whiskers belong to the water balance estimates. Most of the displayed theoretical curves (Table 1) that predict E/P as a function of Φ are parameter-free; where they are not, a value that provided the best overall fit by eyeballing was chosen. E_0 is estimated by the Priestley-Taylor equation with $\alpha = 1.31$; E_{PM} is estimated with the Rome wind function.

[9] From Figure 3 it is evident that the data points deviate markedly from the classical Budyko curve. Possible reasons for this have already been discussed by *Budyko* [1974], *Milly* [1994], and more recently by *Donohue et al.* [2007].

[10] The current AA model–based formulation of the water-energy balance can be related to the mean soil water balance of *Porporato et al.* [2004] by writing the last equation of Table 1 as [*Porporato et al.*, 2004]

$$E/P = 1 - \Phi\left[g^{g/\Phi-1}\exp(-g)\right]\left[\Gamma\left(g\Phi^{-1}\right) - \Gamma\left(g\Phi^{-1},g\right)\right]^{-1} = \left[\Phi^{-1} - Nh^{-1}g^{-1}\exp(-g)\right]\Phi, \qquad (4)$$

where the normalization constant, N, is defined as

$$N = hg^{k/h}\left[\Gamma\left(kh^{-1}\right) - \Gamma\left(kh^{-1},g\right)\right]^{-1}. \qquad (5)$$

Here $h = E_0/w_0$ is the normalized evaporation loss under humid conditions, w_0 the maximum soil water storage available to plants; $g = w_0/b$ a dimensionless number with b denoting the mean water depth of (exponentially distributed) rainfall values above a set threshold and the resulting (Poisson-distributed) rainfall events occurring with frequency k; Γ is the complete gamma function with one parameter and the incomplete gamma function with two as defined by *Abramowitz and Stegun* [1964]. After equating (2) and (4) one obtains

$$2 - E_{PM}/E_0 = kh^{-1}g^{-1} - Nh^{-1}g^{-1}\exp(-g) = m_x, \qquad (6)$$

where m_x is the long-term mean of the effective relative soil moisture, x, defined as $(s - s_w)(s_R - s_w)^{-1}$, s and s_w denoting the actual and wilting point soil moisture, respectively, while s_R the runoff (and deep percolation) producing moisture content. The combination of (4), (6), and the relationship $E = m_x E_0$ [*Porporato et al.*, 2004] yields

$$2 - E_{PM}/E_0 = P\left\{1 - \Phi\left[g^{g/\Phi-1}\exp(-g)\right]\left[\Gamma\left(g\Phi^{-1}\right) - \Gamma\left(g\Phi^{-1},g\right)\right]^{-1}\right\}/E_0, \qquad (7)$$

which, employing $g = 9$, results in $R^2 = 0.78$ for the 75 pairs of m_x values, the left side having an ensemble average of 0.72, while the right side average is 0.75.

[11] To the best of our knowledge such a linking of the effective relative soil moisture to different types of the potential evaporation rates has not been reported. Furthermore, w_0, the maximum soil water storage available to

Table 1. Theoretical Curves of E/P as a Function of the Aridity Index, Φ

Equation	References
$1 - \exp(-\Phi)$	*Schreiber* [1904]
$\Phi \tanh(\Phi^{-1})$	*Oldekop* [1911]
$(1 + \Phi^{-n})^{-1/n}$	*Mezentsev* [1955], *Choudhury* [1999], *Yang et al.* [2008], and *Pike* [1964][a]
$\{\Phi \tanh(\Phi^{-1})\,[1 - \exp(-\Phi)]\}^{0.5}$	*Budyko* [1958]
$1 - \Phi\,[g^{g/\Phi-1}\exp(-g)]\,[\Gamma(g\Phi^{-1}) - \Gamma(g\Phi^{-1}, g)]^{-1}$	*Porporato et al.* [2004]

[a]With $n = 2$.

plants, can be estimated from precipitation characteristics, i.e., its mean as well as k and b, provided g is known. The latter, however, can be obtained from additional estimates of E_{PM} and E_0 by choosing a g value that best satisfies (4) where E/P comes from (2), therefore relying only on atmospheric measurements, without the need of knowing runoff and the corresponding drainage area.

[12] w_0 is certainly an important parameter in ecohydrological modeling and its potentially varying mean value as the rooting system of the vegetation responds (by changing the mean rooting depth) to shifts in the temporal distribution of precipitation can this way be predicted solely from atmospheric variables. From the $w_0 = (s_R - s_w)pZ_r$ relationship [*Porporato et al.*, 2004], where p is the soil porosity, Z_r the rooting depth, changes in the latter as a response to climate variations can, in theory, also be predicted from atmospheric measurements of air pressure, temperature, humidity, precipitation, incoming solar (or global) radiation, and wind velocity.

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