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Baseflow separation based on analytical solutions of the Boussinesq equation

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Abstract

A technique for baseflow separation is presented based on similarity solutions of the Boussinesq equation. The method makes use of the simplifying assumptions that a horizontal impermeable layer underlies a Dupuit aquifer which is drained by a fully penetrating stream. The value of the baseflow maximum as well as the baseflow recession hydrograph can be estimated by fitting the analytical solutions of the Boussinesq equation to the observed discharge values for individual flood events. For the rising limb of the baseflow hydrograph a linear function is assumed for simplicity. The method is first demonstrated on three watersheds for a total of five flood events and extensively applied over a three year period for the Mahantango Creek, Pennsylvania watershed. The proposed method reduces some of the subjective aspects long associated with baseflow separation techniques. © 1998 Elsevier Science B.V.

Keywords: Baseflow separation; Baseflow hydrograph; Stream–aquifer interactions

1. Introduction

Baseflow separation from streamflow hydrographs has long been a topic of interest in hydrology (see Hall, 1968, and Tallaksen, 1995, for comprehensive reviews) since the baseflow recession curve itself contains valuable information about the aquifer properties. Baseflow recession analyses are routinely used in low flow forecasting, water supply allocation, hydroelectric powerplant designs and in waste dilution schemes (Tallaksen, 1995). Also, baseflow separation from quick storm response is required for numerous widely used hydrological models (e.g. HEC-1 flood hydrograph package by the US Army Corps of Engineers, unit hydrograph techniques) and other water resource applications (Vogel and Kroll, 1996). The large number of existing techniques and the high level of subjectivity in separating baseflow contribution from total streamflow (Tallaksen, 1995) indicates that the problem is not fully understood. In this paper we propose a baseflow separation technique, applicable for individual flood events, with analytical solutions of the Boussinesq equation. The proposed technique is based on the governing equation for flow in saturated porous media, as opposed to empirical relationships, and reduces the number of subjective elements generally associated with the application of baseflow separation methods.

The one-dimensional Boussinesq equation, when the effect of capillarity above the water table is neglected and the Dupuit–Forcheimer approximation is invoked, describes the elevation of the transient groundwater table $h(x,t)$ above a horizontal

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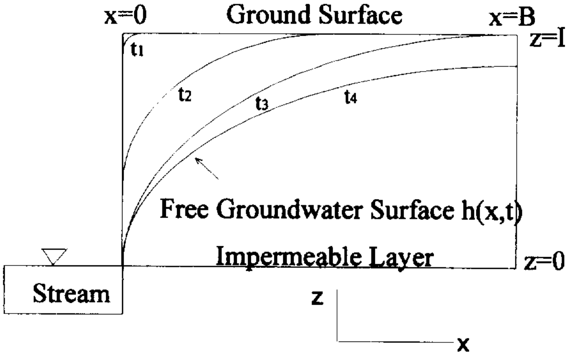


Fig. 1. Schematic diagram of an initially saturated unconfined aquifer with a fully penetrating stream. The recession of the groundwater table shown through time (t).

impermeable layer

$$\frac{\partial h}{\partial t}=\frac{k}{\varphi}\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) \tag{1}$$

where k is the saturated hydraulic conductivity of the unconfined aquifer, φ is the drainable porosity, t is time, and x is horizontal distance. For a so-called fully penetrating stream ($h(0,t) = 0$) draining an initially saturated aquifer ($h(x,0) = D$, where D is the aquifer depth) of finite width (B) (see Fig. 1), the resulting outflow rate q (per unit length) to the channel for a short time (i.e. the drainage is not yet influenced by the no-flow condition at the boundary) is (Brutsaert and Nieber, 1977)

$$q(t)=0.332(k\varphi)^{1/2}D^{3/2}t^{-1/2} \tag{2}$$

When the recession drawdown extends over the entire breadth of the aquifer (i.e. $h(x,t) < D$, everywhere), the long-time outflow rate becomes (Boussinesq, 1903; Polubarinova-Kochina, 1962, p. 517)

$$q(t')=\frac{0.862kD^2}{B\left[1+1.115\left(\frac{kD}{\varphi B^2}\right)t'\right]^2} \tag{3}$$

Note that at the start of the long-time solution the time reference is reset (i.e. $t' = 0$ at time corresponding to the transition from short- to long-time solution). The time origin (i.e. the time when the groundwater drawdown begins) for the short-time solution Eq. (2) is generally not known, so Brutsaert and Nieber (1977) suggested that one could analyze the slope of the recession hydrograph (dQ/dt) as a function of the discharge (Q). For both the short-time (Eq. (2)) and the long-time (Eq. (3)) solutions, the slope of the recession can be expressed as

$$\frac{dQ(t)}{dt}=-aQ^b(t) \tag{4}$$

where Q ($= 2\int_L q\,dl$, where L is the total length of the contributing channels) is the measured discharge, and a and b are constants. With appropriate expressions for the drainage density R_d ($= LA^{-1}$, where A is the area of the watershed) and an effective width B ($=(2R_d)^{-1}$) for natural watersheds, the constants in Eq. (4) are,

$$a_1=\frac{1.133}{k\varphi D^3L^2},\quad b_1=3 \tag{5}$$

for the short-time solution, and,

$$a_2=\frac{4.804k^{1/2}L}{\varphi A^{3/2}},\quad b_2=\frac{3}{2} \tag{6}$$

for the long-time solution (Brutsaert and Nieber, 1977; Troch et al., 1993). Eq. (4) plotted as $\log(-dQ/dt)$ *versus* $\log(Q)$ forms two straight lines with slopes of 3 and 1.5, and intercepts a_1 and a_2, corresponding to the solutions Eqs. (5) and (6), respectively. In Fig. 2 the short- and long-time solutions Eqs. (2) and (3) are plotted.

The general solution of Eq. (4) with initial condition $Q(0) = Q_0$, the discharge at the (unknown) time origin, is

$$Q(t)=\left(Q_0^{1-b}-(1-b)at\right)^{\frac{1}{1-b}},\ \text{if } b\neq 1;$$
$$Q(t)=Q_0e^{-at},\ \text{if } b=1 \tag{7}$$

(see e.g. Singh, 1988). Eqs. (4)–(7) form the basis for separating the baseflow from the total runoff during storm events in this paper.

2. Study site description

The proposed methodology for baseflow separation is applied to four watersheds in the United States. Two regionally representative watersheds (B and F) are in the Little River (a tributary to the Suwanee River) basin near Tifton, Georgia. The third catchment used is known as watershed 522 of the Washita

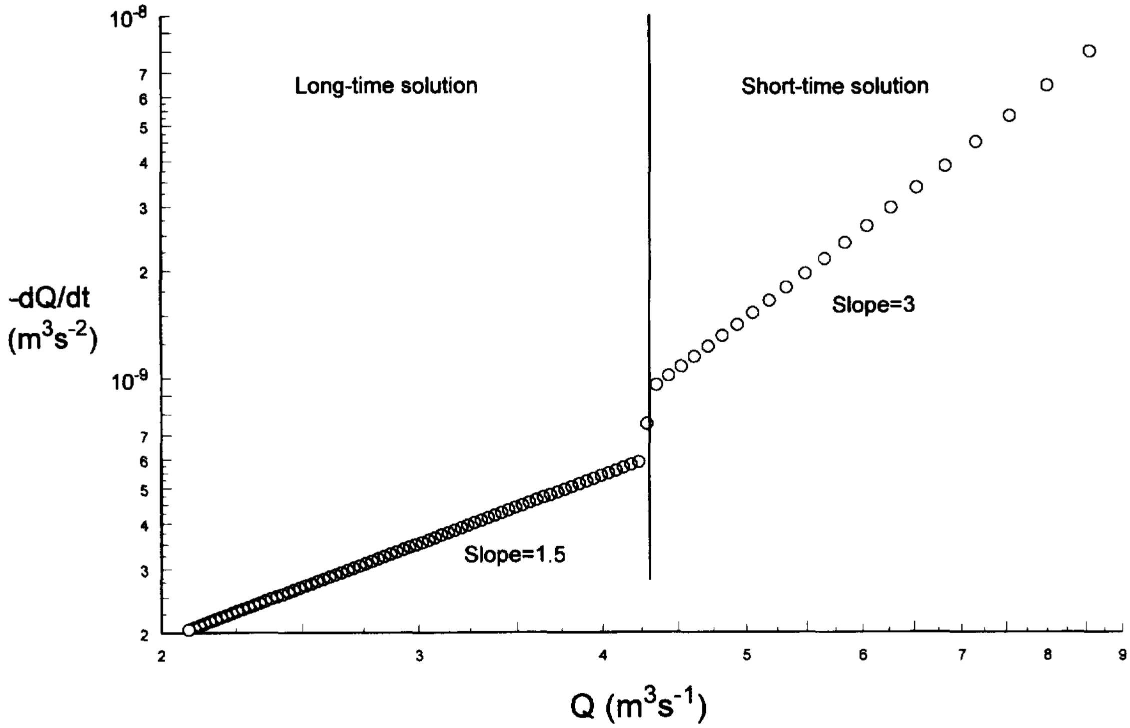


Fig. 2. The time rate of change in discharge versus magnitude of discharge for the analytical solutions of the one-dimensional Boussinesq equation.

River (Chickasha) Experimental Watershed near Minnekah in Oklahoma. This is also considered to be a regionally representative catchment. The discharge measurements for individual storm events were selected from the USDA, Miscellaneous Publications 1164, 1446, 1453 and 1469, that describe regionally representative experimental agricultural watersheds in the US. The fourth watershed is the Mahantango Creek watershed, which is a tributary of the Susquehanna River in Pennsylvania.

Watershed B in Georgia has an area of 334 km^2. It is covered with sediments of early to middle Miocene age that are underlain by limestones. The sediments are composed of poorly-sorted sands interbedded with partly-indurated sandy claystones and clays. The streams within the watershed are incised into these materials which inhibit deep seepage loss. The entire area is underlain by the Hawthorn Formation, which is considered an aquiclude. The annual loss of groundwater to deep seepage is less than 25 mm per year. The surface soil is permeable and, in general, the infiltration rates are high. Of the watershed area, 40% is forest, 36% is crop, 18% is pasture, and the rest is miscellaneous. The drainage density is 1.9 km^{-1}, the mean annual precipitation is 1295 mm which is distributed evenly throughout the year. Due to the precipitation pattern, high infiltration rates and relatively shallow soil depth conditions (1–3 m), groundwater represents a steady contribution to channel runoff and accounts for about 40–60% of total runoff. During flood events water stages were recorded by two Fisher and Porter digital stage recorders up- and downstream of a broad-crested V-notch weir with 10 to 1 side slopes (USDA, Miscellaneous Publication 1464).

Watershed F is a sub-basin of watershed B with an area of 114 km^2. The soil and geologic conditions are the same as for watershed B where 44% of the watershed is covered by forest, 33% is cropland and 17% is pasture. Discharge flow rates were measured the same way as in case of watershed B. The drainage density is 2.7 km^{-1} (USDA, Miscellaneous Publication 1469).

Watershed 522 in Oklahoma has an area of

539 km^2. It is covered with deep sandy (5–15 m) and moderately deep loamy soils (3–10 m) derived from sandstone on a gently rolling landscape. The land use is classified as 66% of the catchment is pasture and rangeland, 17% is crop, and 16% is miscellaneous (farmsteads, roads, airports, etc.). The discharge rates were derived from stage measurements by means of a rating curve (USDA, Miscellaneous Publication 1446). The drainage density of the catchment is not published. The mean annual precipitation is 762 mm with the highest monthly sums in early summer. Baseflow contribution to total runoff is generally small, it is around 10% of the annual discharge value.

Mahantango Creek is a tributary of the Susquehanna River, located in the non-glaciated part of the North Appalachian Ridge and Valley Region. Going from northwest to southeast the catchment is characterized by Devonian sandstone, siltstone, and shale (USDA, Miscellaneous Publication 1453) underlying thin moderately weathered channery or stony loam soils with poorly developed horizons. The catchment has an area of 423 km^2 and a drainage density of 0.68 km^{-1}. The precipitation is distributed evenly throughout the year with a mean annual total of some 1000 mm. Baseflow is about 30% of the total runoff. The catchment is heavily forested, more than 70% of its area is woodland. Water stages were recorded hourly and transformed into discharge values using a rating curve for the period between 1 October 1993 and 25 November 1996.

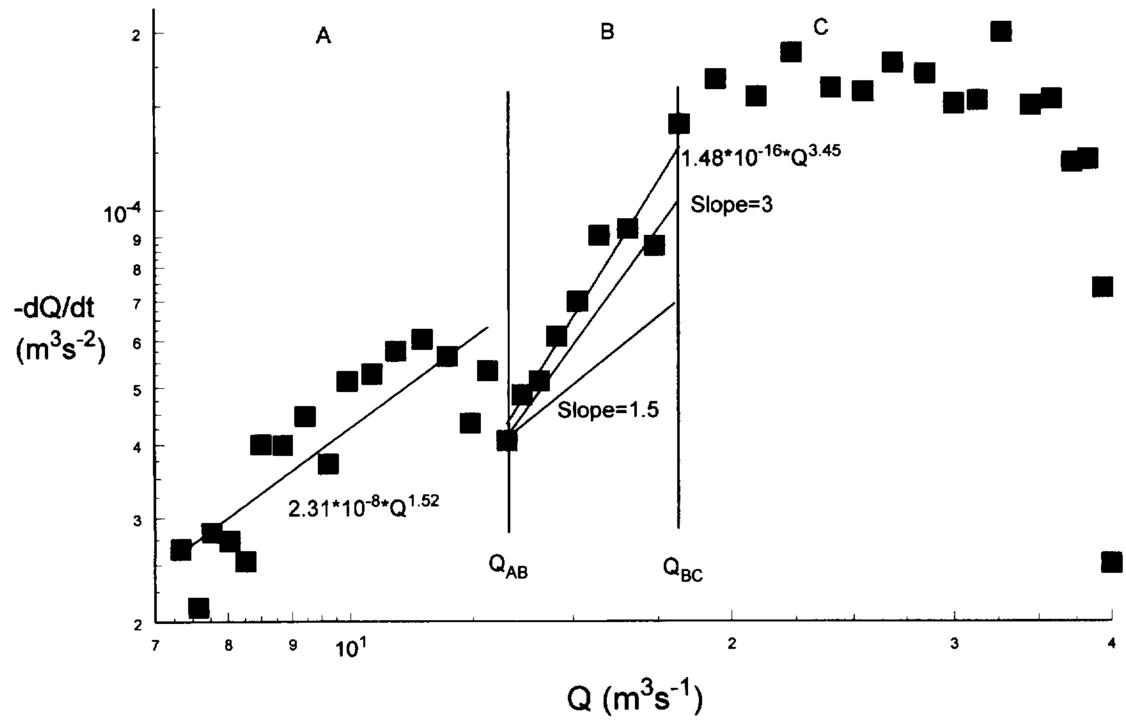
3. Data sets

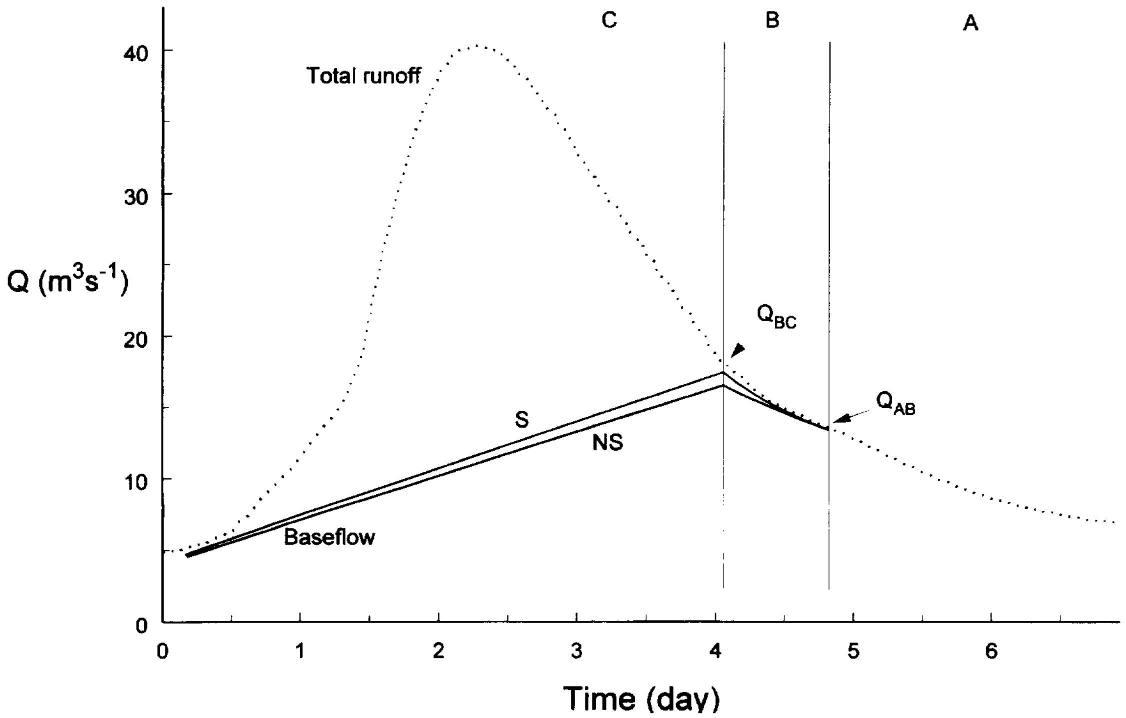
We selected two flood events (in the years 1970 and 1978) for watershed F in Georgia, two for watershed B (1972, 1973) and one (1975) for watershed 522 in Oklahoma. For the receding limb of each flood hydrograph we plotted the $\log(-\Delta Q/\Delta t)$ *versus* $\log(Q)$ values (applying a central difference scheme), where t was set to be a constant with a value between 2.5 and 5 h, depending on the available mean interval length between adjacent discharge measurements, for a particular flood event. We note here that the choice of the time step t is somewhat arbitrary, although, in principle the results should not depend on its value. However, when t is chosen to be too small, the relative importance of measurement errors in the discharge values are amplified since the $\Delta Q/\Delta t$ values may decrease to a point comparable to measurement uncertainty. When Δt is too large the temporally variable role of the different processes contributing to total runoff are blended together and more difficult to separate. As a compromise a Δt interval of a couple of hours was used in this study.

The time-interval lengths between discharges published depend on the discharge rate changes (except for Mahantango Creek) and they vary between 5 min and a couple of hours. The chosen values of Δt make it possible to have enough points for the analysis, but at the same time help reduce measurement errors in the values of $\Delta Q/\Delta t$. Discharge rates between actual measurements were estimated using a linear interpolation before calculating the $\Delta Q/\Delta t$ values which allowed use of a constant Δt to analyze a particular flood event. The sections of the recession hydrographs where Δt was less than the time interval between adjacent measurements (at the very end of some of the storm events) were not included in the recession flow analysis. This was less than 5% of the data available for the five flood events.

For the Mahantango Creek watershed we use a constant 3 h interval throughout the measurement period of more than 3 years. The falling limbs of the hydrographs were automatically selected from the discharge

Fig. 3. (a) The time rate of change in discharge versus discharge rate. Recession hydrograph, catchment B, Little River watershed near Tifton, Georgia, event of 13–19 January 1972. Time flows from right to left. The different sections of the graph are: (A) pure baseflow region, characterized by the smallest changes in discharge rates; (B) transient region, characterized by the steepest positive slope; (C) quick storm response region, characterized by the largest changes in discharge rates plus a very steep negative slope to the rightmost part of the region corresponding to the acceleration of the drainage rate changes. The best fit equations for regions A and B are shown as well as the short- (slope = 3) and long-time (slope = 1.5) analytical solutions of the Boussinesq equation going through the discharge value (Q_{AB}) separating the two regions. (b) Observed total runoff and estimated baseflow hydrographs. Catchment B, Little River watershed near Tifton, Georgia, event of 13–19 January 1972. S, baseflow hydrograph using the short-time solutions in region B; NS, only the long time solution is used. In region A the observed runoff is assumed to be pure baseflow.


A
B
C
$1.48*10^{-16}*Q^{3.45}$
Slope=3
Slope=1.5
$2.31*10^{-8}*Q^{1.52}$
Q_{AB}
Q_{BC}
-dQ/dt (m^3s^{-2})
Q (m^3s^{-1})


C
B
A
Total runoff
Q_{BC}
Q_{AB}
S
NS
Baseflow
Q (m^3s^{-1})
Time (day)

record whenever the recession lasted longer than 45 h without any break. This was necessary in order to provide enough data points for the proposed baseflow separation technique for each individual flood hydrograph. Eventually 27 flood events were obtained and analyzed for the Mahantango Creek watershed.

4. Proposed baseflow separation

The receding limb of the hydrograph reflects the rate at which water is drained from the catchment. Since the rate of groundwater depletion is less than the rate at which other sources of the total runoff are drained (i.e. surface flow, subsurface stormflow, interflow, and channel storage), the milder slope (i.e. the tail) of the recession hydrograph is generally attributed entirely to groundwater drainage (Barnes, 1939; 1940). This groundwater drainage can be described by Eq. (4), as was presented by Brutsaert and Nieber (1977). Plotting $\log(-dQ/dt)$ against $\log(Q)$ under conditions required for the analytical solutions of the Boussinesq equation one may expect to observe a slope of either 3 or 1.5, according to Eq. (5) or Eq. (6), for the baseflow recession hydrograph.

We now present the proposed baseflow separation in Fig. 3a and b for a January flood event on catchment B, Little River watershed near Tifton, Georgia. First we plot $\log(-dQ/dt)$ *versus* $\log(Q)$ to study the behavior of Eq. (4) with Eqs. (5) and (6). In Fig. 3a a slope of 1.5 is clearly observable (as illustrated by the best fit line equation) at the leftmost part (region A) of the log–log graph where the receding limb of the runoff hydrograph is plotted. It is not unreasonable to assume that in this region of the hydrograph the observed runoff might well be pure baseflow (i.e. groundwater drainage) since it closely matches Eq. (4) in combination with Eq. (6).

In region B a slope somewhat steeper than 3 (illustrated by the best fit line equation) is found in the graph. As was discussed above, the short-time solution Eq. (2) of the Boussinesq equation corresponds to a slope of 3 on the double-logarithmic graph. However, Verma and Brutsaert (1971) showed that at early times of the groundwater recession "the effect of partly saturated or capillary flow is quite important, so that the outflow decreases faster than it would if the groundwater table were a true free surface". If the watershed became saturated during the precipitation event then in region B one might expect to find the short-time response of the groundwater drainage that can be estimated by drawing a line with slope 3 starting at Q_{AB} (i.e. the minimum discharge value in the steepest slope region) in the double-logarithmic graph and extending it up to Q_{BC} (i.e. the maximum discharge value in the region with the steepest slope). The difference between the measured $\Delta Q/\Delta t$ values and the ones represented by the line drawn, is due in part to capillary and partly saturated flow and/or the residual effect of other quick storm response components (i.e. overland flow, interflow, quick subsurface flow, and channel storage) (Brutsaert and Nieber, 1977).

In region A the runoff apparently can be assumed to be pure baseflow, while in region B some effect of the quick storm response is seen in the recession hydrograph, however, runoff is still dominated by baseflow.

Notice that region C is characterized by the highest (almost constant) rate of change in discharge. This is clearly the region where quick storm response dominates the streamflow generating mechanisms (Verma and Brutsaert, 1971; Brutsaert and Nieber, 1977; Parlange et al., 1989). In the rightmost part of region C of the double-logarithmic graph there is a very steep descent (going from left to right) that corresponds to the convex part of the original $Q(t)$ recession hydrograph. In this part of region C the drainage rate increases rapidly with time as the portion of the watershed that contributes to runoff does. The largest rate of change in discharge corresponds to the inflexion point of the runoff hydrograph marking the time (i.e. the so-called time of concentration) when the total area of the watershed contributes to the observed runoff (Viessman et al., 1989).

Since the groundwater-exchange to the stream responds more slowly to precipitation events than quick storm response does (hence the name), one might logically assume that the quickest changes expressed in the recession hydrograph (i.e. region C) correspond to a situation where quick storm transport of effective precipitation is dominant and the groundwater is still mainly being replenished. Furthermore, since region B with a slope close to 3 corresponds to the short-time solution of the Boussinesq equation, it is felt that the baseflow maximum could be where region B and C meet.

The actual baseflow recession curve can be obtained by transforming the lines, corresponding to the analytical solutions of the Boussinesq equation in region B of the double-logarithmic graph, into $Q(t)$ values. This can be achieved by applying Eq. (7) with $b = 3$ and $Q_0 = Q_{AB}$ (see Fig. 3a), and propagating the solution backwards in time. The value of a (i.e. the intercept) can be calculated from Eq. (4) with Q_{AB} and the corresponding $-\Delta Q/\Delta t$ values. The baseflow maximum results at time equal to the elapsed time between the observed Q_{AB} and Q_{BC} discharge values. (Notice, that the best fit line equations for region A and B are never used in the analysis, their role is only to illustrate the slope of the log–log graph in those regions).

Fig. 3b displays the estimated baseflow hydrograph. The approach so far has only provided the baseflow recession curve and nothing concerning the rising limb of the baseflow hydrograph. In the absence of that information we simply connected the base of the total runoff hydrograph and the estimated baseflow peak with a straight line. Notice that the use of the short- *versus* long-time solution in region B results in practically the same baseflow maxima (the same is true for all flood events investigated, as shown in Table 1), thus for practical applications the use of only the long-time solution is satisfactory. Notice also that in Fig. 3b a small bump can be found (at around Q_{AB}) in the otherwise smooth runoff recession curve. This region might be attributed to bank-storage effects and manifest itself as a region with reversed slope (due to the water, stored in the stream banks, being discharged into the channel and thus representing an extra—though short-lived—source for groundwater drainage) in the corresponding double-logarithmic graph (see e.g. Linsley et al., 1958; Singh, 1968; Raudkivi, 1979).

5. Results and discussion

The proposed baseflow separation, demonstrated in Fig. 3a and b, was first applied for four additional storm events and extended over a three year period of hourly data for the Mahantango Creek watershed in Pennsylvania. Table 1 summarizes the characteristics of the observed total runoff and estimated baseflow recession hydrographs for watersheds in Georgia and Oklahoma. Note that the estimated baseflow peak

Table 1
Characteristics of the total runoff and estimated baseflow recessions of individual flood events

	Precipitation sum (mm)	Peak discharge ($m^3 s^{-1}$)	Time to peak discharge (days)	Estimated baseflow peak ($m^3 s^{-1}$)	Estimated time to baseflow peak (days)	Estimated discharge when surface runoff becomes zero ($m^3 s^{-1}$)	Estimated time when surface runoff becomes zero (days)
TiftonWatershed FArea = 114 km^2	46	42	1.0	9(S) 8(NS)	1.9	7	3.4
	69	47	1.3	12(S) 11(NS)	2.4	9	2.9
TiftonWatershed BArea = 334 km^2	89	40	2.2	17(S) 16(NS)	4	13	4.8
ChickashaWater-shed 522Area = 539 km^2	137	58	2.7	18(S) 17(NS)	4.75	13	5.4
	56	126	0.3	19(S) 17(NS)	0.6	13	1.1

S, applying short-time (region B) plus long-time solutions (region A); NS, applying only the long-time solution.

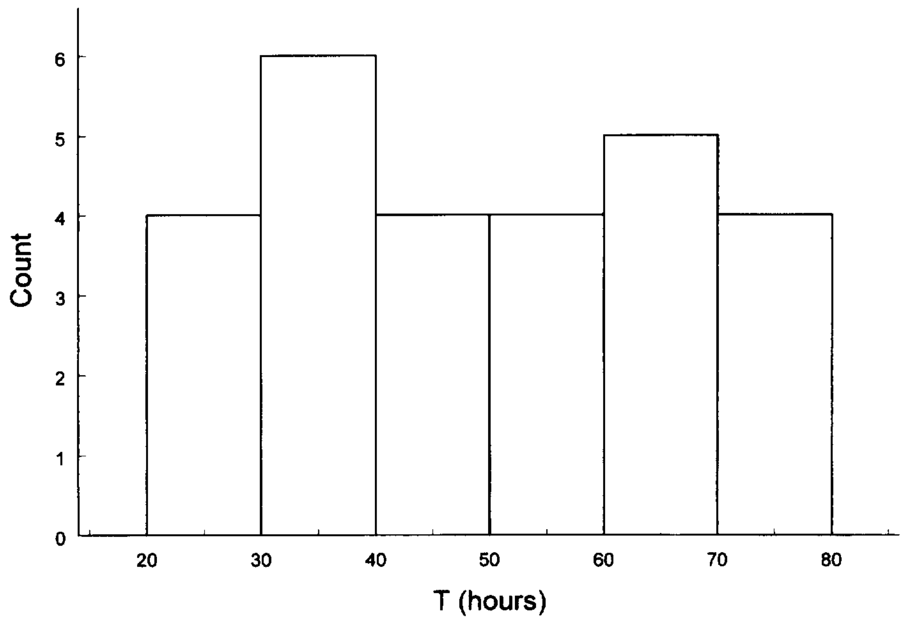


Fig. 4. Histogram of elapsed time (T) between the peak of total runoff and the beginning of estimated pure groundwater flow for 27 flood events, Mahantango Creek, Pennsylvania.

within the same catchment is in correlation with the magnitude of the triggering precipitation. This is so partly because the antecedent precipitation indices were practically zero (USDA, Miscellaneous Publications 1164, 1446, and 1469). Table 1 reveals that the time to peak (i.e. the time elapsed from the beginning of the rising limb of the hydrograph to the peak discharge) for the estimated baseflow component (within the same watershed) was approximately twice as large as the time to peak for the total runoff.

The so-called watershed specific time intervals (T) between the peak of total runoff and the beginning of pure groundwater flow from Table 1 (after taking averages where necessary) match (except for watershed 522) with those of the so called Area Method introduced by Linsley et al. (1958). Our estimated values are 2.0, 2.65, and 0.8 day, while the same estimates with the area method ($T = A^{0.2}$, where the watershed area, A, is measured in square miles) are 2.13, 2.64, and 2.9, respectively. The relatively large difference in the watershed specific time interval for the Oklahoma catchment might be explained if we assume (without proof) that the precipitation event was not strong enough to recharge the entire watershed due to its deep soils, but rather only in areas where the groundwater is closer to the surface (i.e. near the channel). It means that the value of A in the Area Method should be reduced, however to an unknown extent.

Fig. 4 displays the histogram of the T values for the Mahantango catchment based on the analysis of 27 flood events where the recession hydrographs were automatically selected from the hourly discharge record of more than three years and the double logarithmic graphs were drawn. After a visual inspection of each graph the Q_{AB} values could be identified and the corresponding T values were calculated automatically using only the long-time solution of the Boussinesq equation. While the Area Method gives a value of 66 h for T, our analysis resulted in a wide range of T values with a mean of 48 h suggesting that T is a function of actual antecedent hydrologic conditions of the catchment.

Fig. 5 displays the time rate of change in discharge *versus* discharge rate for the Mahantango Creek catchment based on daily mean discharge data taken at least 5 days after the cessation of precipitation in the period between 1984 and 1987. A slope of 1.5 of the 97% lower envelope indicates the applicability

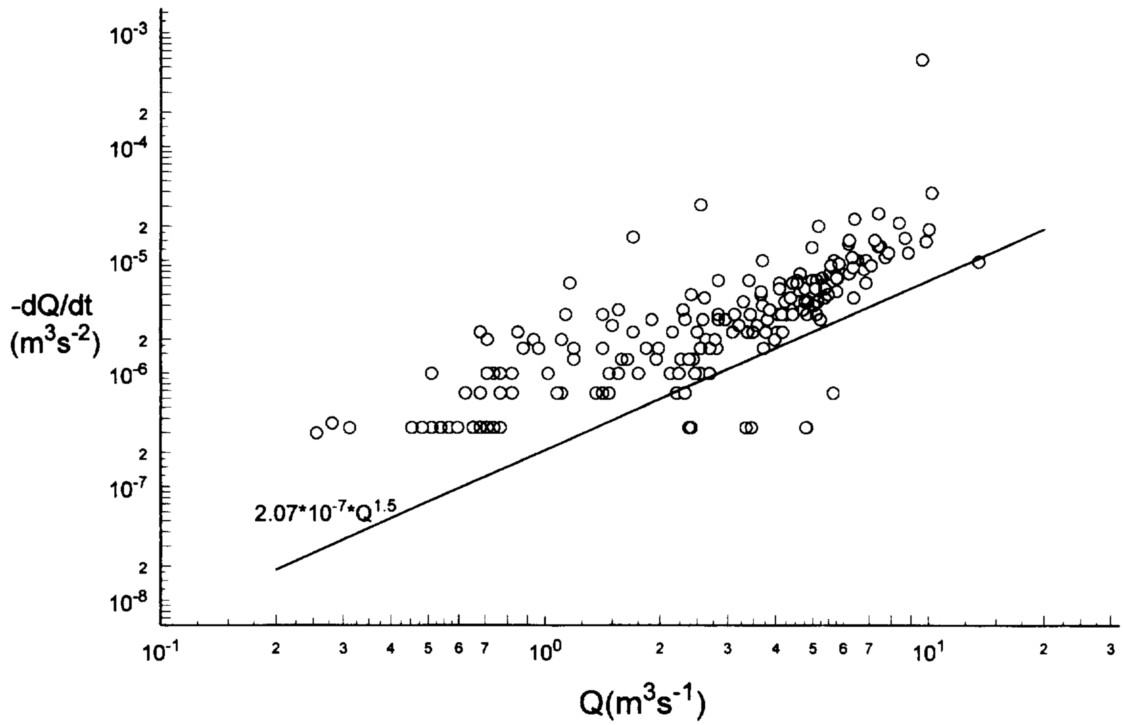


Fig. 5. The time rate of change in discharge rates versus discharge rates, Mahantango Creek, Pennsylvania. The daily mean discharges were taken at least 5 days after any rainfall had taken place in the period between 1984 and 1987. The line is the 97% lower envelope.

of the long-time solution of the Boussinesq equation as was first proposed by Brutsaert and Nieber (1977).

6. Summary

The proposed baseflow separation technique involves the following steps:

1. Plot $\log(-\Delta Q/\Delta t)$ *versus* $\log(Q)$ for the receding limb of the flood hydrograph with an appropriate value for Δt.
2. Identification of region A at low discharge values characterized with a slope of 1.5.
3. Identification of region B to the right of region A with a slope steeper than 1.5. Draw a straight line with a slope of 1.5 through the smallest discharge value (i.e. Q_{AB}) in region B and extend it up to region C (i.e. to Q_{BC}, the maximum discharge value in the steepest slope region). Region C contains the highest rates of change in runoff values.
4. Transform the straight line in region B into Q(t) values by the application of Eqs. (4) and (7). The baseflow maximum results at the backward propagated time equaling the elapsed time between the observed Q_{AB} and Q_{BC} discharge values.
5. Draw a straight line between the beginning of the rising hydrograph and the estimated baseflow maximum. This part, by default, is arbitrary.

In summary, the above described steps of baseflow separation are easily transformable into an automated computer algorithm as was partially demonstrated for the Mahantango catchment. The technique was applied to four watersheds and for 32 flood events in total in order to identify the baseflow component of the runoff hydrograph and resulted in estimated baseflow parameters comparable to those of a more empirical and commonly used method (i.e. Area Method). The proposed technique is based on the analytical solutions of the Boussinesq equation and provides the user with the estimated baseflow hydrograph.

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