**Water Resources Research**

**COMMENT**
10.1029/2018WR023502

This article is a comment on Han and Tian (2018), https://doi.org/10.1029/2017WR021755.

**Key Points:**
- The exact functional form of the nondimensional complementary relationship (CR) of evaporation remains unknown.
- Application of a sigmoid function leads to physical contradictions.
- The slope of the CR function at the upper boundary must be constrained by the slope of the Priestley-Taylor line limit

**Correspondence to**
J. Szilagyi, jszilagyi1@unl.edu

**Citation:**

Received 14 JUN 2018
Accepted 2 JAN 2019
Accepted article online 8 JAN 2019

---

**Comment on “Derivation of a Sigmoid Generalized Complementary Function for Evaporation With Physical Constraints” by S. Han and F. Tian**

Jozsef Szilagyi1,2 and Richard D. Crago1

1Conservation and Survey Division, School of Natural Resources, University of Nebraska-Lincoln, Lincoln, NE, USA.
2Department of Hydraulic and Water Resources Engineering, Budapest University of Technology and Economics, Budapest, Hungary.
3Department of Civil and Environmental Engineering, Bucknell University, Lewisburg, PA, USA

**Abstract** The sigmoid function Han and Tian derive for their $E/E_p = f(E_r/E_p)$ complementary relationship leads to physical contradictions, therefore cannot be accepted as an improvement of existing complementary relationship theory.

Han and Tian attempt to derive a new complementary relationship (CR) between two nondimensional variables, $x = E_r/E_p$ and $y = E/E_p$, in the form of a preconceived sigmoid shape. Here $E$ is the actual, while $E_p (= E_r + E_a)$ the Penman-derived (Penman, 1948) potential evaporation rate made up of the $E_r$ energy and $E_a$ aerodynamic terms, such as

$$E_r = \frac{\Delta (R_n - G)}{\Delta + \gamma}, \quad E_a = \frac{\gamma f_u d_s}{\Delta}.$$  \hspace{1cm} (1)

where $\Delta$ is the slope of the saturation vapor pressure curve, $\gamma$ the psychrometric constant, $R_n$ the surface net radiation, $G$ the ground heat flux, $f_u$ the wind function, and $d_s$ the vapor pressure deficit. Following Crago et al. (2016) and Szilagyi et al. (2017), Han and Tian set a constant lower limit, $x_{min}$, for $x$, although this lower limit clearly changes with the measurement period (Crago et al., 2016) and is definitely not a constant in time. Similarly, they set another constant value, $x_{max}$, for $x$ when the $E_a$ term reaches its minimum. Additionally, they introduce two more parameters for regulating the shape of the sigmoid function.

Let us note that the $x_{max}$ value, when $E_a$ is minimal under wet conditions (i.e., when water availability for evaporation is nonrestricting on a regional scale), takes up the role of the Priestley and Taylor (1972) parameter, $\alpha$, that is, $E_{p \text{ wet}} = \alpha E_r + E_a$; $\alpha = E_r/x_{max} - E_a$, $\therefore x_{max} = \alpha^{-1}$.

The upper boundary condition (BC) proposed by Han and Tian, namely, that $dy/dx = 0$ at $y = 1$, creates a hard to interpret physical situation. Due to the flat upper part of their CR curve, $E$ remains equal to $E_p$ for a while as the $x$ value starts to decrease from its $x_{max}$ value. In reality, however, with the region drying out, $E$ can be expected to decrease from its maximum value of $E_p = E_{p \text{ wet}}$, attained under regionally wet conditions. The $E = E_p$ condition for $x$ close to $x_{max}$ (the result of the flat upper portion of Han and Tian’s curve) entails that $E$ increases in the beginning of the drying process under a constant $E_r$ term, since the $E_a$ term of $E_p$ must increase to be able to move $x$ from its maximum value of $x_{max}$. As a result, the model-derived $E$ not only increases with the drying out of the environment but one also ends up with a $y$ value above the $E_{p \gamma}$ limit line ($y_{max}$), which thus loses its constraining property. Note that with $x_{max} = \alpha^{-1}$, point $M$ of the $E_{p \gamma}$ line in Figure 1 of Han and Tian (which corresponds to $x = 1.26$ in their figure) moves right horizontally, to be situated above $x_{max}$ and by doing so crosses the solid red line of the sigmoid function to terminate on it (see Figure 1 below).

Let us see now how the authors arrived at their BC of (i) $dy/dx = 0$ at $y = 1$. From partial derivatives of the $E/E_p = f(E_r/E_p)$ CR equation with respect to $E_r$ and $E_a$, Han and Tian correctly obtained two solutions, (i) and (ii) $\partial E_r/\partial E_p = 1 = \alpha - 1$, here written with the $x_{max} = \alpha^{-1}$ substitution. One can also obtain solution (ii) by applying the derivation with respect to $E_r$ directly on $E_a^{\min} = (\alpha - 1)E_r$ and additionally assuming that $\alpha
is independent of $E_s$. However, they discard solution (ii) by saying that $\alpha$ must also depend on $E_s$. Such a direct dependence, however, has not been shown in the literature, even though air temperature (among other variables) may be an influencing factor on the value of $\alpha$, as Han and Tian correctly quote.

So instead of only (i), (ii) may also be a solution to their system of equations, which means, at the very least, that there exists another solution beside $dy / dx = 0$ at $y = 1$. In fact, due to the physical controversies this latter solution presents, one must conclude that there is only one physically interpretable solution, and that is solution (ii). Solution (ii) however does not restrict the value of $dy / dx$ to zero at $y = 1$, thus a $dy / dx$ value of $\alpha$ at $y = 1$, first proposed by Brutsaert (2015), is perfectly acceptable, as it fully avoids the physical contradictions raised by the $dy / dx = 0$ value of Han and Tian.

In summary it can be stated that the sigmoid function Han and Tian propose for the CR relationship leads to physical contradictions, the result of their improper upper BC. This way one cannot consider their study an improvement upon recent CR studies by Brutsaert (2015), Crago et al. (2016), and Szilagyi et al. (2017).

References