Water Resources Research

COMMENT
10.1029/2018WR023502

This article is a comment on Han and Tian (2018), https://doi.org/10.1029/2017WR021755.

Key Points:
- The exact functional form of the nondimensional complementary relationship (CR) of evaporation remains unknown
- Application of a sigmoid function leads to physical contradictions
- The slope of the CR function at the upper boundary must be constrained by the slope of the Priestley-Taylor limit line

Correspondence to:
J. Szilagyi,
jszilagyi1@unl.edu

Citation:
Szilagyi, J., & Crago, R. D. (2019). Comment on "derivation of a sigmoid generalized complementary function for evaporation with physical constraints" by S. Han and F. Tian. *Water Resources Research*, *55*. https://doi.org/10.1029/2018WR023502

Received 14 JUN 2018
Accepted 2 JAN 2019
Accepted article online 8 JAN 2019

Comment on "Derivation of a Sigmoid Generalized Complementary Function for Evaporation With Physical Constraints" by S. Han and F. Tian

Jozsef Szilagyi[1,2] **and Richard D. Crago**[3]

[1]Conservation and Survey Division, School of Natural Resources, University of Nebraska-Lincoln, Lincoln, NE, USA, [2]Department of Hydraulic and Water Resources Engineering, Budapest University of Technology and Economics, Budapest, Hungary, [3]Department of Civil and Environmental Engineering, Bucknell University, Lewisburg, PA, USA

Abstract The sigmoid function Han and Tian derive for their $E/E_p = f\,(E_r/E_p)$ complementary relationship leads to physical contradictions, therefore cannot be accepted as an improvement of existing complementary relationship theory.

Han and Tian attempt to derive a new complementary relationship (CR) between two nondimensional variables, $x = E_r/E_p$ and $y = E/E_p$, in the form of a preconceived sigmoid shape. Here E is the actual, while E_p $(= E_r + E_a)$ the Penman-derived (Penman, 1948) potential evaporation rate made up of the E_r energy and E_a aerodynamic terms, such as

$$E_r = \frac{\Delta(R_n - G)}{\Delta + \gamma}, \qquad E_a = \frac{\gamma f_u d_v}{\Delta + \gamma}, \tag{1}$$

where Δ is the slope of the saturation vapor pressure curve, γ the psychrometric constant, R_n the surface net radiation, G the ground heat flux, f_u the wind function, and d_v the vapor pressure deficit. Following Crago et al. (2016) and Szilagyi et al. (2017), Han and Tian set a constant lower limit, x_{min}, for x, although this lower limit clearly changes with the measurement period (Crago et al., 2016) and is definitely not a constant in time. Similarly, they set another constant value, x_{max}, for x when the E_a term reaches its minimum. Additionally, they introduce two more parameters for regulating the shape of the sigmoid function.

Let us note that the x_{max} value, when E_a is minimal under wet conditions (i.e., when water availability for evaporation is nonrestricting on a regional scale), takes up the role of the Priestley and Taylor (1972) parameter, α, that is, $E_p^{\text{wet}} = \alpha E_r = E_r + E_a^{min} = E_r + E_r/x_{max} - E_r$, thus $x_{max} = \alpha^{-1}$.

The upper boundary condition (BC) proposed by Han and Tian, namely, that $dy/dx = 0$ at $y = 1$, creates a hard to interpret physical situation. Due to the flat upper part of their CR curve, E remains equal to E_p for a while as the x value starts to decrease from its x_{max} value. In reality, however, with the region drying out, E can be expected to decrease from its maximum value of $E_p = E_p^{wet}$, attained under regionally wet conditions. The $E = E_p$ condition for x close to x_{max} (the result of the flat upper portion of Han and Tian's curve) entails that E increases in the beginning of the drying process under a constant E_r term, since the E_a term of E_p must increase to be able to move x from its maximum value of x_{max}. As a result, the model-derived E not only increases with the drying out of the environment but one also ends up with a y value above the E_{PT} limit line ($=\alpha x$), which thus loses its constraining property. Note that with $x_{max} = \alpha^{-1}$, point M of the E_{PT} line in Figure 1 of Han and Tian (which corresponds to $\alpha = 1.26$ in their figure) moves right horizontally, to be situated above x_{max}, and by doing so crosses the solid red line of the sigmoid function to terminate on it (see Figure 1 below).

Let us see now how the authors arrived at their BC of (i) $dy/dx = 0$ at $y = 1$. From partial derivatives of the $E/E_p = f\,(E_r/E_p)$ CR equation with respect to E_r and E_a, Han and Tian correctly obtained two solutions, (i) and (ii) $\partial E_a/\partial E_r|_{y=1} = \alpha - 1$, here written with the $x_{max} = \alpha^{-1}$ substitution. One can also obtain solution (ii) by applying the derivation with respect to E_r directly on $E_a^{min} = (\alpha - 1)\,E_r$ and additionally assuming that α

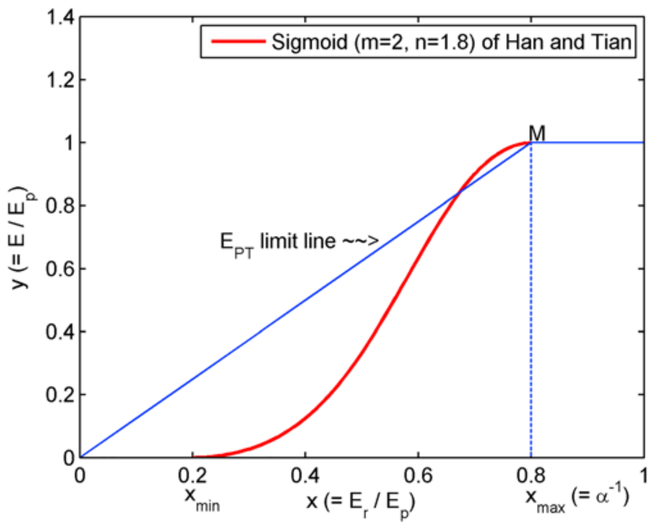

Figure 1. Relative positions of the E_{PT} limit line and the sigmoid function recommended by Han and Tian. The sigmoid function will always be above the limit line as x nears x_{max}. Here an $\alpha = 1.26$ value was used for the illustration, but α can take up any value typically from the [1–1.32] interval; thus, the slope of the E_{PT} limit line changes accordingly.

is independent of E_r. However, they discard solution (ii) by saying that α must also depend on E_r. Such a direct dependence, however, has not been shown in the literature, even though air temperature (among other variables) may be an influencing factor on the value of α, as Han and Tian correctly quote.

So instead of only (i), (ii) may also be a solution to their system of equations, which means, at the very least, that there exists another solution beside $dy\ /\ dx = 0$ at $y = 1$. In fact, due to the physical controversies this latter solution presents, one must conclude that there is only one physically interpretable solution, and that is solution (ii). Solution (ii) however does not restrict the value of $dy\ /\ dx$ to zero at $y = 1$, thus a dy/dx value of α at $y = 1$, first proposed by Brutsaert (2015), is perfectly acceptable, as it fully avoids the physical contradictions raised by the $dy/dx = 0$ value of Han and Tian.

In summary it can be stated that the sigmoid function Han and Tian propose for the CR relationship leads to physical contradictions, the result of their improper upper BC. This way one cannot consider their study an improvement upon recent CR studies by Brutsaert (2015), Crago et al. (2016), and Szilagyi et al. (2017).

Acknowledgments
This work was supported by the BME-Water sciences & Disaster Prevention FIKP Grant of EMMI (BME FIKP-VÍZ).

References

Brutsaert, W. (2015). A generalized complementary principle with physical constraints for land-surface evaporation. *Water Resources Research*, *51*, 8087–8093. https://doi.org/10.1002/2015WR017720

Crago, R., Szilagyi, J., Qualls, R. J., & Huntington, J. (2016). Rescaling of the complementary relationship for land surface evaporation. *Water Resources Research*, *52*, 8461–8471. https://doi.org/10.1002/2016WR019753

Han, S., & Tian, F. (2018). Derivation of a Sigmoid Generalized Complementary Function for Evaporation With Physical Constraints. *Journal of Geophysical Research: Water Resources Research*, *54*, 5050–5068. https://doi.org/10.1029/2017WR021755

Penman, H. L. (1948). Natural evaporation from open water, bare soil, and grass. *Proceedings of the Royal Society of London*, *A193*, 120–146.

Priestley, C. H. B., & Taylor, R. J. (1972). On the assessment of surface heat flux and evaporation using large-scale parameters. *Monthly Weather Review*, *100*(2), 81–92. https://doi.org/10.1175/1520-0493(1972)100<0081:OTAOSH>2.3.CO;2

Szilagyi, J., Crago, R., & Qualls, R. J. (2017). A calibration-free formulation of the complementary relationship of evaporation for continental-scale hydrology. *Journal of Geophysical Research: Atmospheres*, *122*, 264–278. https://doi.org/10.1002/2016JD025611