Periodica Polytechnica
Civil Engineering

On the Clark Unit Hydrograph Model of HEC-HMS

Volume (2017) paper 11141
https://doi.org/10.3311/PPci.11141

József Szilágyi[1*]

RESEARCH ARTICLE

Received 21 April 2017; Revised 02 June 2017; Accepted 23 June 2017

Abstract
A mathematically correct form of the linear storage element's routing equation is specified requiring only a slight modification of the existing HEC-HMS routine. The new formulation, unlike the current version in use, is unconditionally stable as long as the time step and the storage coefficient values are trivially specified as positive numbers.

Keywords
linear storage element, unit hydrograph, Muskingum method

1 Department of Hydraulic and Water Resources Engineering
Faculty of Civil Engineering,
Budapest University of Technology and Economics
H-1111 Budapest, Hungary
* Corresponding author email: jszilagyi1@vit.bme.hu

1 Introduction

Application of the HEC-HMS software [1] for watershed modeling is gaining worldwide momentum ([2]–[6]). It is believed that by demonstrating how one of its routines, the Clark Unit Hydrograph Model, can be made more efficient in a simple manner is a worthwhile effort to engage in as it is expected that once these slight changes are incorporated into the model by the United States Army Corps of Engineers (USACE), it will serve the hydrologic modeling and practicing engineering community well, especially as HEC-HMS can be easily coupled to atmospheric [3] or evapotranspiration [7] models, which thus motivated the present work.

Clark [8] derived the unit hydrograph of linear watershed response to excess precipitation via the Muskingum channel routing [9] analogy by considering the inhomogeneous ordinary differential equation (ODE) of the lumped continuity

$$dS(t)/dt = I(t) - Q(t) \tag{1}$$

where dS/dt is the time-rate of change in stored water volume (S) within the catchment (or within the channel reach), I is excess precipitation for the watershed (or inflow rate to the channel), Q is stream discharge at the outlet (or outflow from the channel) and t is time-reference. The Muskingum method relates storage to a weighted average of the in- and outflow rates of the channel section

$$S(t) = K\left[xI(t) + (1-x)Q(t)\right] \tag{2}$$

where K is a constant storage coefficient and $0 \le x \le 1$. By replacing the time rate of change with finite differences for Δt time increments and taking arithmetic averages for $I(t)$ and $Q(t)$ from consecutive values separated by Δt in time, the well-known Muskingum routing equation results [8] as

$$Q_t = c_0 I_t + c_1 I_{t-\Delta t} + c_2 Q_{t-\Delta t} \tag{3}$$

with

$$c_0 = -\left(Kx - 0.5\Delta t\right)/\left(K - Kx + 0.5\Delta t\right) \tag{4}$$

$$c_1 = \left(Kx + 0.5\Delta t\right)/\left(K - Kx + 0.5\Delta t\right) \tag{5}$$

$$c_2 = \left(K - Kx - 0.5\Delta t\right) / \left(K - Kx + 0.5\Delta t\right) \tag{6}$$

$$1 = c_0 + c_1 + c_2 \tag{7}$$

By consideration that precipitation is typically represented as a constant (in the form of pulses) over each Δt, i.e., $I_t = I_{t-\Delta t}$ for the given time period and taking $x = 0$ which thus represents a linear storage element where outflow is proportional to storage, Eq. (3) can be written [1] as

$$\begin{aligned} Q_t &= \left[1 - \Delta t / \left(K + 0.5\Delta t\right)\right] Q_{t-\Delta t} + \left[\Delta t / \left(K + 0.5\Delta t\right)\right] I_t \\ &= \left(1 - c_A\right) Q_{t-\Delta t} + c_A I_t . \end{aligned} \tag{8}$$

Note that here I_t is the constant pulse value valid for the (t–Δt, t) interval. As the coefficient c_A depends on Δt and K as well, $c_A \leq 1$ is required in order to avoid possible negative outflows which thus yields $\Delta t/K \leq 2$ as a general requirement for numerical stability [9].

Stability issue only emerges because of the unnecessary introduction of finite differences for obtaining the routing scheme of Eq. (8). As it is shown below, a very similar routing scheme can be derived (only the calculation of c_A changes) without resorting to finite differences thus avoiding any stability problems in future applications.

2 A mathematically correct formulation of the routing scheme of Equation (8) for the linear storage element

Solution of Eq. (1) for the linear storage element, i.e., $S(t) = KQ(t)$, can be obtained from [11] where a generalized solution for a homogeneous cascade (i.e. K is the same for each linear storage element of the cascade) of serially connected linear storage elements is given by explicitly taking into account how input (inflow or precipitation) and output (flowrate) are represented by measurements in a discrete-time framework. For a detailed description of the mathematics involved please also refer to [12]. The solution for the single linear storage element forced by pulsed precipitation excess values thus becomes

$$S_t = e^{-k\Delta t} S_{t-\Delta t} + K\left(1 - e^{-k\Delta t}\right) I_t \tag{9}$$

with $k = K^{-1}$. Note again that I_t is the constant pulse value valid for the (t–Δt, t) interval. From Eq. (9) the outflow rates become

$$Q_t = e^{-k\Delta t} Q_{t-\Delta t} + \left(1 - e^{-k\Delta t}\right) I_t \tag{10}$$

which is similar to Eq. (8) except [1 – Δt (K + 0.5Δt)–1] is replaced by $e^{-k\Delta t}$ and Δt (K + 0.5Δt)–1 by (1 – $e^{-k\Delta t}$). Note that Eq. (10) can never yield negative values as $e^{-k\Delta t}$ is always less than unity and larger than zero for any positive k and Δt values. This is the mathematically correct formulation of the linear storage element's routing scheme specifically formulated for pulsed inputs in a discrete-time framework.

Fig. 1 illustrates the unconditionally stable nature of Eq. (10) in stark contrast to Eq. (8), the latter becoming increasingly unstable (outflow rates not only exceed concurrent excess precipitation values but also become negative) as $\Delta t/K$ increases.

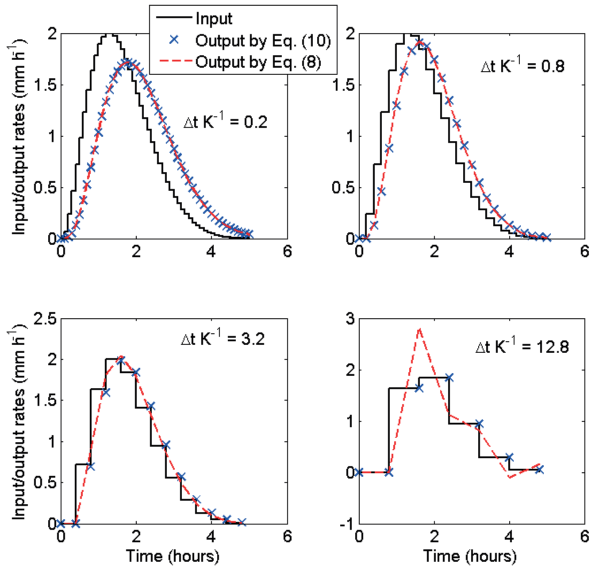

Fig. 1 The results of routing schemes Eqs. (8) and (10) to pulsed precipitation excess values (see [15] for a detailed description of the input time series). With each panel the values of both Δt and K^{-1} are doubled, starting from 0.1 h and 2 h^{-1}, respectively.

3 Conclusions

Eq. (9) is the general solution for storage [and (10) for outflows] of the inhomogeneous bulk mass-conservation ODE represented by (1) and applied for a linear reservoir. As it is unconditionally stable, it is always recommended over the HEC-HMS routing scheme of Eq. (8).

The first term (which contains the instantaneous impulse response function of the linear storage element evaluated at $t = \Delta t$) specifies S at t from an initial value at t–Δt (unforced system response) while the second term (containing the unit-step response function evaluated at $t = \Delta t$) yields the storage response at t to external forces (forced response) represented as excess precipitation pulses. In the classical hydrological literature typically only this forced response of an originally relaxed system [i.e. $S(0) = 0$] is discussed in the form of convolution integrals. Note that by taking into account the pulsed nature of the excess precipitation function the convolution integral can be brought into the explicit form found in Eq. (9). All this is deemed worthwhile to mention as the work of [11], pioneering a generalized state-space description of the classical and still widely used Nash cascade [13] of serially connected linear storage elements, is largely overlooked in the hydrologic and civil engineering literature leading to repeated claims of generalization (e.g., [14]) by inclusion of the unforced response.

References

[1] US Army Corps of Engineers Hydrologic Engineering Center (USACE) “Hydrologic modeling system HEC-HMS user’s manual.” http://www.hec.usace.army.mil/software/hec-hms/ [Accessed: 5, April 2017].

[2] Olivera, F. “Extracting hydrologic information from spatial data for HMS modeling”. *Journal of Hydrologic Engineering.* 6(6), pp. 524-530. 2001. https://pdfs.semanticscholar.org/c2a5/7016d8dd1c4414ee-0a5e29a5f60da535c189.pdf

[3] Anderson, M. L., Chen, Z-Q., Kavvas, M. L., Feldman, A. “Coupling HEC-HMS with atmospheric models for prediction of watershed runoff”. *Journal of Hydrologic Engineering.* 7(4), pp. 312-318. 2002. https://doi.org/10.1061/(ASCE)1084-0699(2002)7:4(312)

[4] Yilmaz, A. G., Imteaz, M. A., Ogwuda, O. “Accuracy of HEC-HMS and LBRM models in simulating snow runoffs in Upper Euphrates Basin”. *Journal of Hydrologic Engineering.* 17(2), pp. 342-347. 2012. https://doi.org/10.1061/(ASCE)HE.1943-5584.0000442

[5] Gyawali, R., Watkins, D. W. “Continuous hydrologic modeling of snow-affected watersheds in the Great Lakes Basin using HEC-HMS”. *Journal of Hydrologic Engineering.* 18(1), pp. 29-39. 2013. https://doi.org/10.1061/(ASCE)HE.1943-5584.0000591

[6] De Silva, M. M. G. T., Weerakoon, S. B., Herath, S. “Modeling of event and continuous flow hydrographs with HEC–HMS: case study in the Kelani River Basin, Sri Lanka”. *Journal of Hydrologic Engineering.* 19(4), pp. 800-806. 2014. https://doi.org/10.1061/(ASCE)HE.1943-5584.0000846

[7] Szilagyi, J., Kovacs, A., “Complementary-relationship based evapotranspiration mapping (CREMAP) technique for Hungary”. *Periodica Polytechnica Civil Engineering.* 54(2), pp. 95-100. 2010. https://doi.org/10.3311/pp.ci.2010-2.04

[8] Clark, C. O. “Storage and the unit hydrograph”. *Transactions of the American Society of Civil Engineers*. 110(1), pp. 1419-1446. 1945.

[9] McCarthy, G. T. “*The unit hydrograph and flood routing*”. Unpublished paper, Conference of the North Atlantic Division, USACE, New London, CN. 1938.

[10] Ponce, V. M. “*Engineering hydrology: principles and practices*”. Prentice Hall, Englewood Cliffs, NJ, USA. 1989.

[11] Szollosi-Nagy, A. “The discretization of the continuous linear cascade by means of state-space analysis”. *Journal of Hydrology.* 58(3-4), pp. 223-236. 1982. https://doi.org/10.1016/0022-1694(82)90036-1

[12] Szilagyi, J., Szollosi-Nagy, A. “*Recursive streamflow forecasting: a state-space approach*”. CRC Press, Leiden, The Netherlands. 2010. https://doi.org/10.1201/b10538

[13] Nash, J. E. “The form of the instantaneous unit hydrograph”. *International Association of Hydrological Sciences*. 45(3-4), pp. 114-121. 1957. http://iahs.info/uploads/dms/045011.pdf

[14] Yan, B., Guo, S., Liang, J., Sun, H. “The generalized Nash model for river flow routing”. *Journal of Hydrology,* 530, pp. 79-86. 2015. https://doi.org/10.1016/j.jhydrol.2015.09.055

[15] Szilagyi, J. ‘‘Accounting for stream-aquifer interactions in the state-space discretization of the Kalinin-Milyukov-Nash cascade for streamflow forecasting’’. *Journal of Hydrologic Engineering.* 9(2), pp. 135–143. 2004. https://doi.org/10.1061/(ASCE)1084-0699(2004)9:2(135)