On the Use of Semi-Logarithmic Plots for Baseflow Separation

by Jozsef Szilagyi[a]

Abstract

Recession flow has long been considered a composite of exponential terms, where each exponential term represents a different source of water discharged from the watershed. The changing slopes in the semi-logarithmic plot of the discharge have been considered indicative of the decreasing contribution of surface runoff and interflow to the discharge. The results of this analysis show that the changing slope in the recession plot can be the consequence of baseflow drainage. This can invalidate the semi-logarithmic baseflow separation technique when applied to some hydrologic settings.

Introduction

The use of semi-logarithmic plots for graphical baseflow separation was first introduced by Barnes (Barnes 1939) and has since been used extensively in the hydrologic literature (see Tallaksen [1995] for a comprehensive review; also Feldman [1995], and more recently Martin and Lavabre [1997]). Recession flow in this approach is considered a superposition of individual exponential terms, which supposedly represent different components (i.e., surface, unsaturated, and saturated flow) of the catchment response to precipitation. Since the saturated flow component behaves sluggishly compared with other possible runoff mechanisms, one can argue that after a certain time following the peak of runoff, there remains only one exponential term, representing the ground water contribution to runoff. It has indeed been verified recently in numerous watersheds by Vogel and Kroll (1992) and Brutsaert and Lopez (1998), while in other catchments, no single exponential term could be found (Brutsaert and Nieber 1977; Troch et al. 1993). In a laboratory and in field experiments, Anderson and Burt (1980) checked the validity of the assumptions behind the application of semi-logarithmic graphs for baseflow separation. They demonstrated that the technique indicated false sources, other than ground water, for the observed pure ground water drainage. However, they failed to give a quantitative explanation for the failure of the technique, which motivated the present work.

Theoretical Background

As was suggested by Barnes (1939), the recession limb of a hydrograph can generally be decomposed into three different exponential-type processes:

$$Q = Q_1e^{-t/k_1} + Q_2e^{-t/k_2} + Q_3e^{-t/k_3} \qquad (1)$$

where Q is observed runoff; t is time; and k_1, k_2, and k_3 are the depletion coefficients of surface runoff, interflow, and baseflow, respectively. After some time (i.e., at large times) only the ground water term remains because the magnitude of the depletion coefficients is expected to increase with growing subscript values in Equation 1. Plotting the discharge values on a semi-logarithmic graph against time, one obtains a straight line at large times with a slope of $(-k_3)^{-1}$ in the recession hydrograph. However, moving backward in time on the graph, the slope will increase in magnitude as the effect of the other two terms is increasingly felt. See Singh (1988, page 93) for a graphical example.

The increasing magnitude of the slopes present in a semi-logarithmic plot of the recession flow values does not necessarily mean that the effect of, e.g., surface runoff and/or interflow is being observed, as was experimentally pointed out by Anderson and Burt (1980). In fact, a changing slope in the recession flow values can be shown to be a general property of pure baseflow recession in the following way.

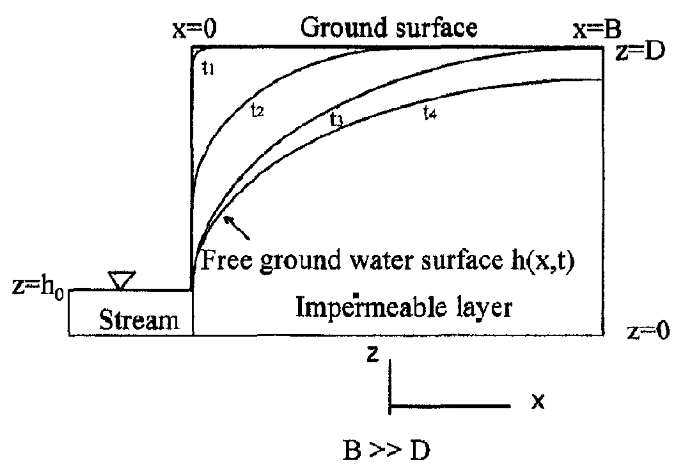

Figure 1. A schematic cross section of the aquifer.

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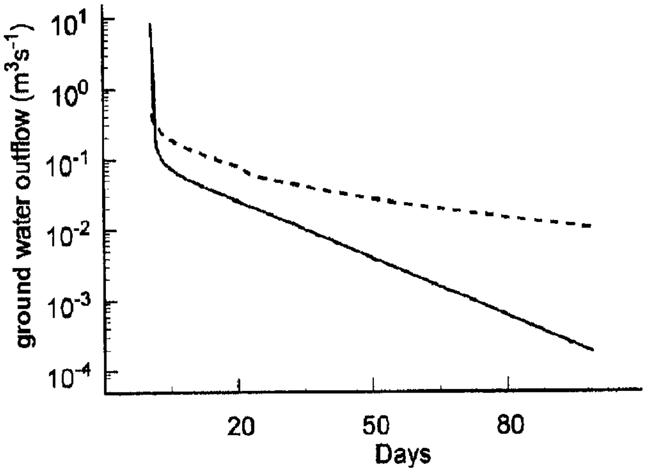

Figure 2. Ground water outflow values in the solution of (a) the linearized Boussinesq equation (solid line); (b) the nonlinear Boussinesq equation (dotted line).

Table 1 Parameter Values Used in Plotting Figure 2 (from Brutsaert and Lopez 1998)	
Saturated hydraulic conductivity (ms^{-1})	.000757
Drainable porosity	.0167
Aquifer breadth (m)	393
Saturated thickness (m)	1.3
Stream level (m) in the analytical solution	1
Stream level (m) in the numerical solution	.1

Neglecting the effect of capillarity, the ground water flow in a rectangular aquifer over a horizontal impermeable layer under Dupuit's condition can be described by the Boussinesq equation (Brutsaert 1994):

$$q = -kh\frac{\partial h}{\partial x} \qquad (2)$$

where $q[L^2/T]$ is the flow rate, h is the water table elevation above the impermeable layer, k is the saturated hydraulic conductivity, and x is the spatial coordinate (Brutsaert 1994). Figure 1 shows a schematic of the system. Equation 2 can be linearized if the elevation of the ground water table changes relatively little in the direction of the flow (Brutsaert and Nieber 1977), resulting in

$$q = -k\rho D\frac{\partial h}{\partial x} \qquad (3)$$

where ρ is a constant that compensates for the approximation introduced by the linearization (Brutsaert 1994). The solution of Equation 3 at x = 0 (i.e., the ground water outflow to the stream) is

$$q = 2k\rho D(D - h_0)B^{-1}\sum_{n=1,2,\ldots}^{\infty}\exp\left[-\frac{(2n-1)^2\pi^2 k\rho D}{4B^2 f}t\right] \qquad (4)$$

with f being the drainable porosity, B the breadth of the aquifer, and h_0 the characteristic value of the water stages in the stream (Brutsaert

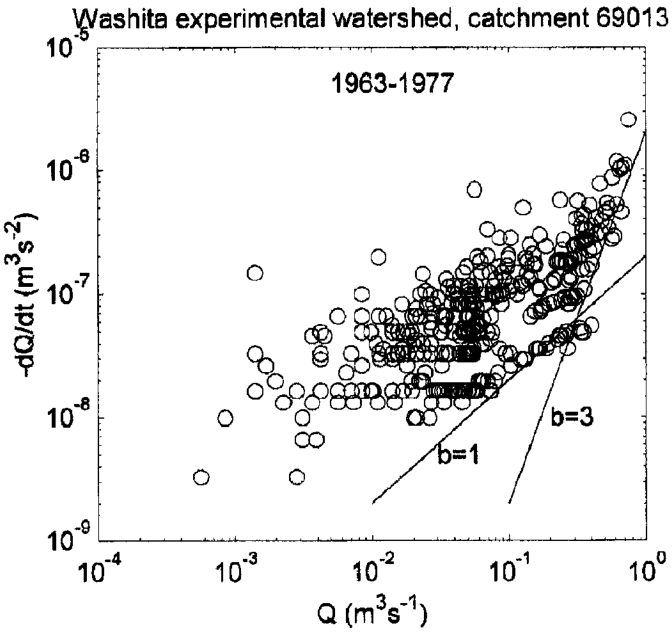

Figure 3. Measured daily discharge versus change in discharge between consecutive days, six days after rain. The lower envelope lines with slopes 1 and 3, respectively, are the long-time and short-time analytical solutions of the linearized Boussinesq equation.

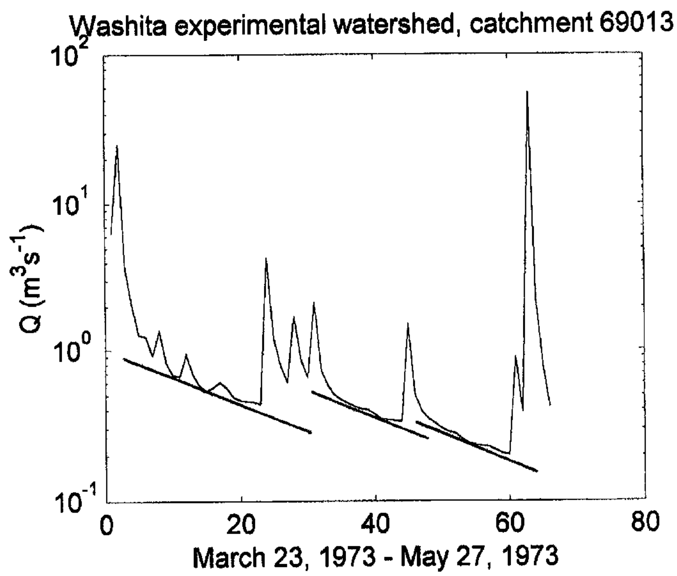

Figure 4. Semi-logarithmic plot of measured daily discharge values. The assumed baseflow recession segments of the hydrograph are marked by straight lines.

and Nieber 1977). Equation 4, when plotted in a semi-logarithmic graph, exhibits ever-changing slopes before it develops a constant slope of $-\pi^2 k\rho D/4B^2 f$ at large times. Figure 2 displays the analytical solution Equation 4 with values (Table 1) of the parameters (i.e., the geometric means of the 22 subbasin-parameters) taken from Brutsaert and Lopez (1998).

Even when the conditions required for the linearization of Equation 1 are not met, the numerical solution of Equation 1 still exhibits changing slopes in the semi-logarithmic plot of the ground water outflow values as demonstrated in Figure 2.

Results

Adopting the superposition theory of the different runoff generation processes in Equation 1, one could conclude that the presence of quick storm response is detectable in the graph (Figure 2), which is clearly erroneous. The implication is that the semi-logarithmic plot itself cannot be conclusive about the source (i.e., whether it is quick storm or baseflow response) of runoff.

To illustrate the problem with a practical example, the measured daily runoff values were plotted in Figure 3, in a double-logarithmic graph, against the changes in discharge between consecutive days, six days after rain. Catchment 69013 is a sub-watershed of the Washita River Experimental Watershed in Oklahoma. The drainage area of the catchment is 154 km^2. For more information on the watershed and measurement techniques applied, see Brutsaert and Lopez (1998). The data points of Figure 3 exhibit two distinct slopes: one of unity and one of 3. The unit slope represents the large-time solution of Equation 3 (Brutsaert and Lopez 1998), which transforms into straight-line segments in the observed hydrograph when plotted in a semi-logarithmic graph. In Figure 4, a couple of these straight-line segments are marked.

If one accepts the assumption, inherent in the application of semi-logarithmic plots for baseflow separation, that baseflow must follow an exponential extinction, then one may falsely conclude that the steepest part of Figure 3 (with a slope of 3) must correspond to quick storm response, instead of the short-time behavior of Equation 3, as was described by Brutsaert and Lopez (1998) using the same data as presented here. From the same faulty argument it further follows that, after six days of rain, quick storm response should still generally be observable. The Area Method (Linsley et al. 1958), however, gives about two days (= N) for the quick storm response to disappear. The Area Method, albeit empirical, has been verified by many authors. For a most recent study, see Szilagyi and Parlange (1998).

Comparing the mean annual volume of pure baseflow (i.e., accounting for days when it can be assumed that runoff is almost entirely supported by ground water) first (case A) assuming that N is about six days (i.e., the case of pure exponential decay); and then (case B) accounting for changing slopes in baseflow response according to Equation 3 and applying an N value of two days, the following characteristics are obtained for the ground water contribution. In case A pure baseflow is present on 27 days a year with an annual mean flow volume of $2.88 \cdot 10^5$ m^3 (3% of the mean annual runoff of $9.32 \cdot 10^6$ m^3), while in case B these values are 69 days and $8.14 \cdot 10^5$ m^3 (9% of the mean annual runoff). Note the differences in these statistics.

While in the described example derived baseflow characteristics differ somewhat, one should be cautious with generalization. In cases where the watershed does not exhibit the short-time behavior of the ground water recession, the two techniques might give similar results.

Conclusions

The application of semi-logarithmic graphs, in the absence of other hydrologic information, to detect different components of the runoff generation process cannot be conclusive due to the changing slopes of the ground water discharge values (when plotted in a semi-logarithmic graph), observable in the solutions of the Boussinesq equation. One might argue that the Boussinesq equation has only limited power to describe complex watersheds due to its inherent simplifying assumptions; consequently its solutions would not be applicable to natural aquifers (Tallaksen 1995). Most recent applications of the analytical solutions of the Boussinesq equation for aquifer characterization (Brutsaert and Lopez 1998; Szilagyi et al. 1998) and baseflow separation (Szilagyi and Parlange 1998), however, seem to corroborate the effectiveness and practical applicability of the technique. It must be acknowledged here that any type of baseflow separation contains some degree of subjectiveness. However, the application of the Boussinesq equation for baseflow separation purposes incorporates the physics of the ground water flow process, thus reducing some of the inherent uncertainties present in other separation techniques and is recommended over traditional hydrograph separation methods, such as the application of semi-logarithmic graphs.

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