ground water

Regional Estimation of Base Recharge to Ground Water Using Water Balance and a Base-Flow Index

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Abstract

Naturally occurring long-term mean annual base recharge to ground water in Nebraska was estimated with the help of a water-balance approach and an objective automated technique for base-flow separation involving minimal parameter-optimization requirements. Base recharge is equal to total recharge minus the amount of evapotranspiration coming directly from ground water. The estimation of evapotranspiration in the water-balance equation avoids the need to specify a contributing drainage area for ground water, which in certain cases may be considerably different from the drainage area for surface runoff. Evapotranspiration was calculated by the WREVAP model at the Solar and Meteorological Surface Observation Network (SAMSON) sites. Long-term mean annual base recharge was derived by determining the product of estimated long-term mean annual runoff (the difference between precipitation and evapotranspiration) and the base-flow index (BFI). The BFI was calculated from discharge data obtained from the U.S. Geological Survey's gauging stations in Nebraska. Mapping was achieved by using geographic information systems (GIS) and geostatistics. This approach is best suited for regional-scale applications. It does not require complex hydrogeologic modeling nor detailed knowledge of soil characteristics, vegetation cover, or land-use practices. Long-term mean annual base recharge rates in excess of 110 mm/year resulted in the extreme eastern part of Nebraska. The western portion of the state expressed rates of only 15 to 20 mm annually, while the Sandhills region of north-central Nebraska was estimated to receive twice as much base recharge (40 to 50 mm/year) as areas south of it.

Introduction

Recharge to ground water is an important variable in regional-scale hydrogeologic models and aquifer-system analysis (Gehrels et al. 2001). While other parts of the water-balance equation, such as precipitation and runoff, are relatively easy to measure, recharge remains an elusive process to quantify. This is especially so because it depends not only on precipitation but also on meteorological conditions, as well as on soil type, soil-moisture status, vegetation cover and condition, slope, cultivation practices, and most of all, on evapotranspiration, which is a function of the previously noted factors.

Currently, standard techniques of estimating regional recharge most often involve (1) applying a water-balance model, where the moisture content of the soil is tracked through time (Vorosmarty et al. 1989; Szilagyi and Vorosmarty 1997; Gehrels et al. 2001), or (2) parameter-value adjustment of ground water flow models (Luckey et al. 1986). Application of the first approach, while generally less intensive computationally, requires knowledge of the vegetation and soil types within the study area, in addition to a number of basic meteorological variables such as air temperature and precipitation. The second approach is more taxing of computer resources because a potentially complex ground water flow model may have to be run repeatedly in search of a multidimensional parameter value optimum.

The proposed approach of base-recharge estimation presented in this paper offers an estimate of total recharge for regions where ground water evaporation is negligible,

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i.e., for areas where the water table is not so close to the surface that the vegetation can use it through its root system. It is computationally simple, requires minimal optimization, and does not need information on vegetation and soil types. The technique is mainly a collection of existing methods which, to the best knowledge of the authors, have not yet been combined in a similar fashion for recharge estimation. It is expected to be most practical for regional-scale studies where the long-term mean annual value of the spatially variable recharge is of interest. The method was tested using data from Nebraska to demonstrate the utility of the technique.

Methodology

The water balance of a geographic region can, in general, be written as

$$P = ET + q_s + q_b + N + \Delta S \tag{1}$$

where P is precipitation (LT^{-1}); ET is evapotranspiration (LT^{-1}); q_s is surface runoff (LT^{-1}); q_b is the ground water contribution to runoff (LT^{-1}), which is the definition of base flow; N is the net flux (LT^{-1}) of any water entering or leaving the region other than precipitation (e.g., water diversions, ground water flux across basin boundaries, and irrigation); and ΔS is the change in stored water (LT^{-1}) within the area. Generally, evapotranspiration is by far the largest loss term in Equation 1, amounting to 70% of precipitation (including evaporation from open water surfaces) on a global basis (Brutsaert 1982). Long-term ET measurements are practically nonexistent, and the available ET estimation methods may differ by as much as 10% to 20% on an annual basis (Vorosmarty et al. 1998). In light of these uncertainties, the general assumption that ΔS is negligible in most cases on a long-term basis may be well justified. For our purposes, this assumption is employed, acknowledging that for some watersheds where hydraulic heads have changed significantly in the past, it may lead to biased recharge estimates. It is further assumed that N in Equation 1 can be neglected as well, at least on a regional scale. In Nebraska, as well as elsewhere, there are watersheds or portions of watersheds where ignoring net fluxes other than precipitation may not be appropriate. Note that this study aims only to map general spatial patterns in naturally occurring base recharge, and is not concerned with human-induced effects such as reservoirs, leaking canals, or irrigation projects. It is felt that this latter, more complicated goal should be the subject of a separate study that could build on the results of this work.

With regard to the stated assumptions, Equation 1 simplifies to

$$P - ET = q_s + q_b \tag{2}$$

which states that the difference between precipitation and ET emerges as surface runoff and base flow. If the change in the stored water volume is negligible, as was assumed, then on a long-term basis, base flow must represent a lower bound to ground water recharge within a given watershed. By quantifying q_b, one obtains an estimate of recharge, provided that the portion of areal ET originating from the ground water is negligible when compared to the total ET of the watershed. In Nebraska, ET from ground water is mainly restricted to areas where phreatophyte vegetation borders higher order streams and to wetland areas, the latter accounting for ~4% of the total land area of the state (Dahl 1990). Although the areal extent of wetlands in Nebraska is small, its effect on recharge may not be negligible due to the relatively large difference between actual ET levels of these wetlands and of other areas. Phreatophyte and wetland vegetation may evaporate at the potential level, which can be three times the level of the ET of nonwetland areas (Nagel and Dart 1980) in Nebraska. Ground water evaporation by phreatophytes and open water evaporation from wetlands must be balanced by increased rates of infiltration, although not necessarily within the same area, if ground water is kept at a dynamic equilibrium over time.

Base flow can be separated automatically from measured discharge values for a given stream by applying a digital filter to the time-series data, as discussed by Nathan and McMahon (1990). Chapman (1991) presented the filter relationship of Nathan and McMahon (1990) in terms of base flow, Q_b (L^3T^{-1}), and total stream discharge:

$$Q_b(i) = kQ_b(i-1) + \frac{1-k}{2}[Q(i) + Q(i-1)] \tag{3}$$

where k is the filter parameter (dimensionless), and Q (L^3T^{-1}) is the measured mean daily stream discharge at day i. The resulting base-flow values are constrained by the concurrent observed stream discharges. Figure 1 illustrates the estimated base-flow component for the Elkhorn River in eastern Nebraska as a result of applying the data-filtering process. This digital filtering algorithm yields similar results to other existing techniques (Arnold and Allen 1999), provided its filter parameter value is chosen correctly. The filter parameter, k, was optimized by systematically changing its value in Equation 3, starting from a minimum trial value of 0.01 and increasing it by 0.01 until $k < 1$, resulting in an optimum of $k = 0.93$ over the gauging stations in Nebraska (Figure 2). With each k value chosen, estimated base-flow time series were created for every gauging station. When the resulting Q_b value was larger than the corresponding measured stream discharge, it was replaced by the measured value. The optimization was based on the watershed-specific time delay (Linsley et al. 1958) given by $N_d = A^{0.2}$, where A is the drainage area for surface runoff above the gauging station, measured in square miles. N_d (in days) is defined as the elapsed time between the peak of the measured discharge and the event when surface runoff is assumed to vanish (i.e., when the value of the simulated base flow first becomes constrained by the measured discharge). The optimized value of k = 0.93 ensures that the resulting N_d estimate is identical *in average* to the N_d value of 4.1 days given by Linsley's equation when calculated with the contributing areas of

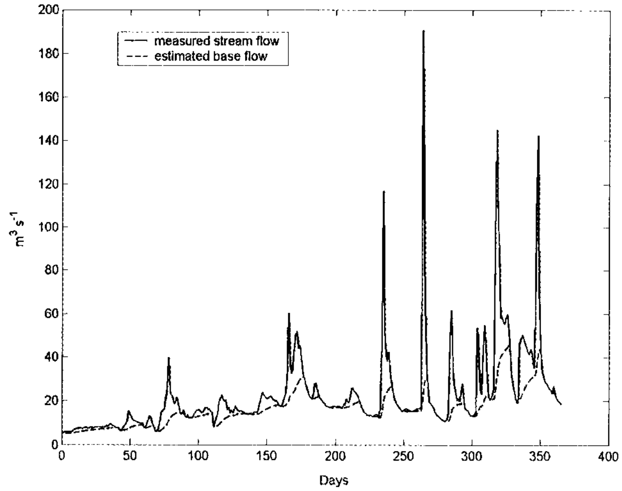

Figure 1. Measured daily mean runoff (solid line) and estimated base-flow discharges, Elkhorn River, West Point, Nebraska, October 1, 1991, through September 30, 1992.

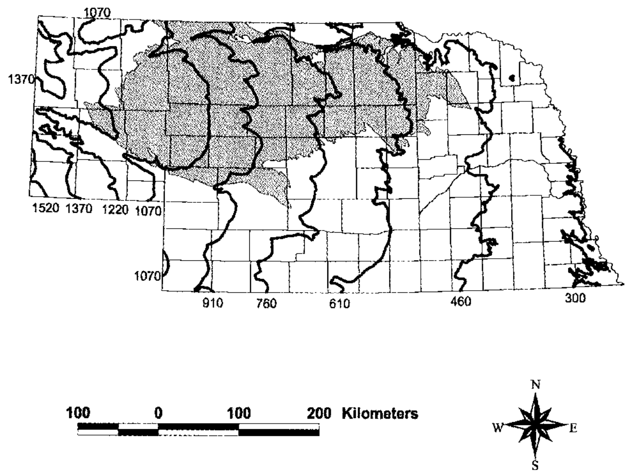

Figure 3. General configuration of the ground water table (m) in Nebraska. The shaded area designates the Sandhills region.

surface water at the 141 gauging stations (Figure 2) in the state.

By noting that base flow remains constant in Equation 3 in the absence of the imposed constraint (i.e., Q_b must not be larger than Q) when measured discharge is supported entirely by ground water (i.e., $Q = Q_b$), Chapman (1991, 1999) later revised the filtering algorithm of Nathan and MacMahon (1990). His revision was aimed to better describe physical properties (i.e., actual base-flow recession); however, we found that an additional constraint (not present in the original form of the filter) was introduced that would not allow base flow to exceed a certain percentage of the measured stream discharge (see Appendix). In the Sandhills region of Nebraska (Figure 3), streamflow is maintained almost entirely by base flow at all times due to the high permeability and hydraulic conductivity of the sandy soils, and so base-flow contribution there far exceeds the maximum allowable 61% by Chapman's algorithm. Consequently, the base-flow separation algorithm (i.e., Equation 3) of Nathan and McMahon (1990) was preferred over Chapman's revision in this study. Contrary to the recommendation of Nathan and McMahon (1990) of applying the filter consecutively in the forward, backward, and (again) forward directions to suppress possible shifts in the base-flow peak, the filter was applied only once with each trial value of k for each discharge time series. The three-pass filtering was considered unnecessary because the exact timing of the base-flow peak is generally unknown (so is any possible shift), and also, because through the selection of a suitable value of k in a single pass of the filter, the estimated base flow can be brought in congruence with one obtained by multiple-pass filtering using a preset value of k.

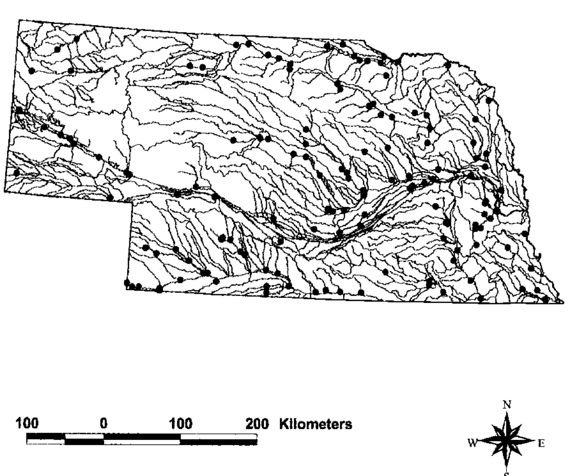

Figure 2. Distribution of the gauging stations in Nebraska, used in the study. The major rivers in the state with their drainage basins are also shown.

The distribution of the U.S. Geological Survey's (USGS) gauging stations with stream discharge records longer than 30 years, where base-flow separation was performed, is shown in Figure 2 for Nebraska. Note that gauging stations downstream of major reservoirs were avoided where it was expected that the reservoir, through its regulated water releases, would have a significant impact on the water regime of the stream, and consequently would corrupt the base-flow separation results.

Base recharge to ground water in units of length over time can, in principle, be obtained by dividing the annual volume of the base flow by the contributing drainage area above a gauging station. However, in Nebraska the contributing drainage area for the ground water is difficult to define because of the general configuration of the ground water table (Figure 3). The water table slopes from west to east across the state with a difference in elevation of ~1200 m. This means that the contributing drainage area for the ground water flow system is likely different than that for surface runoff. A good example may be the case of Frenchman Creek in southwestern Nebraska that once originated near the town of LeRoy in Colorado, 50 km west of the Nebraska border (USDA 1941). Today the river is entirely within Nebraska (NGPC 1983). Because of better water conservation measures and changes in land use (Szilagyi 1999), the contributing area of surface water changed in the past half-century. So did the contributing area of ground water, but most probably at a different rate with a significant decline in ground water levels in the area (CSD 1998) mainly due to large-scale irrigation. Thus today the two contributing areas are probably significantly different, even if they may have been identical before.

As a consequence, the USGS-published drainage area values were used only for the calculation of N_d (where only the drainage area for surface runoff is needed), and instead Equation 2 was employed through the introduction of the dimensionless base-flow index (BFI), which is the ratio of base flow and total stream runoff ($Q = Q_b + Q_s$) over time:

$$BFI = \frac{Q_b}{Q_b + Q_s} \qquad (4)$$

Inserting Equation 4 into Equation 2 yields

$$BFI \times (P - ET) = BFI \times q = q_b \approx R \qquad (5)$$

where R (LT^{-1}) is the yet unknown base recharge, and $q = Q/A_d$, with A_d denoting the contributing drainage area. Note that the automated base-flow separation technique is only used to calculate BFI, but neither q nor q_b were used in Equation 5, because they require the extent of the contributing drainage area, A_d, whereas BFI does not. When the two contributing areas for surface runoff and ground water are known to be fairly close, then q can be used in Equation 5, obliterating the need for the P and ET measurements.

Figure 4 shows the distribution of the climatic stations with long-term daily precipitation values used in the study. Estimates of monthly ET were obtained using the WREVAP model (Morton et al. 1985), one of the most widely tested areal ET estimation tools available (Hobbins et al. 2001). Figure 5 displays the distribution of the Solar and Meteorological Surface Observation Network (SAMSON) stations, where the input variables (precipitation, air temperature and humidity, energy balance) to the WREVAP model were obtained. Precipitation, with the highest spatial variance (Figure 6), is only used to constrain the ET estimates in the WREVAP model. The long-term mean values of temperature, humidity, and the energy balance are expected to change smoothly over the state, so the spatial resolution of the SAMSON network is most probably satisfactory for estimating long-term mean annual ET rates in Nebraska. Note that for the geostatistical generation of the ET surface, not only stations within Nebraska were used but all 22 stations in Figure 5, although with different weights. Hobbins et al. (2001), working from a water-budget analysis of 120 watersheds east of the Continental Divide, showed that the mean difference between the long-term mean annual ET estimates given by WREVAP and the ones given as the difference between precipitation and runoff is practically zero. This difference includes a standard deviation of 6% of the long-term mean annual precipitation in the watershed. This is in keeping with long-term precipitation measurement errors of 5% to 15% (Winter 1981), precipitation being considered the most accurately measurable variable in Equation 1.

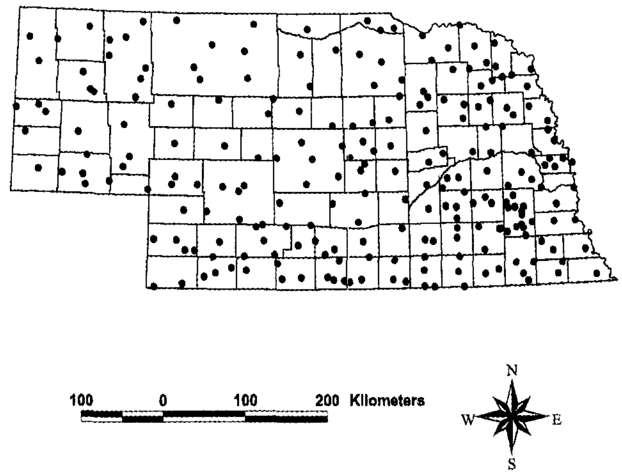

Figure 4. Distribution of the climatic stations in Nebraska with long-term daily precipitation records.

Results and Discussion

From the long-term mean annual values of the point measurements/estimations of P, ET, and BFI, surfaces were generated using ordinary kriging where no apparent spatial drift in the values could be detected (i.e., BFI values), and by using universal kriging with a linear drift otherwise (i.e., with the precipitation and evapotranspiration measurements). Contours of the resulting long-term mean annual P, ET, and BFI fields are shown in Figures 6 through 8. Both P and ET increase from northwest to southeast across the state, the former doubling its long-term mean annual value. Note also that more than 80% of the observed long-term mean annual runoff comes from ground water in the Sandhills region (Figure 3) of north-central Nebraska (Figure 8). The BFI values compare favorably with results of an unpublished work by Bentall and of Bentall and Shaffer (1979) for most part of the state.

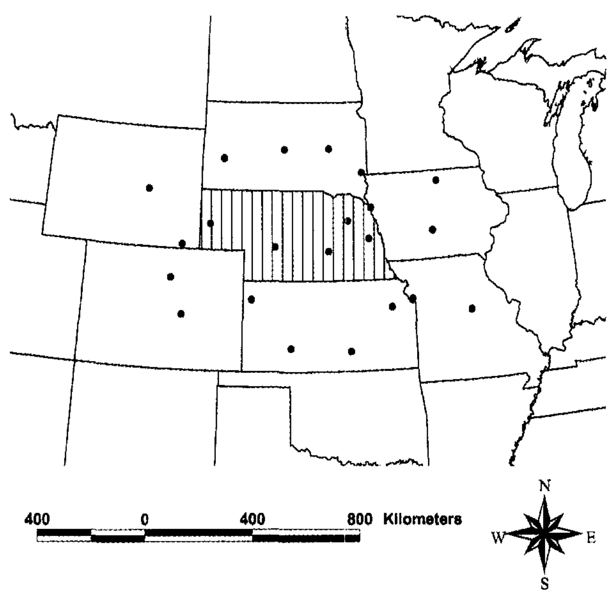

Figure 5. Distribution of the SAMSON stations where long-term mean annual evapotranspiration was calculated by the WREVAP model.

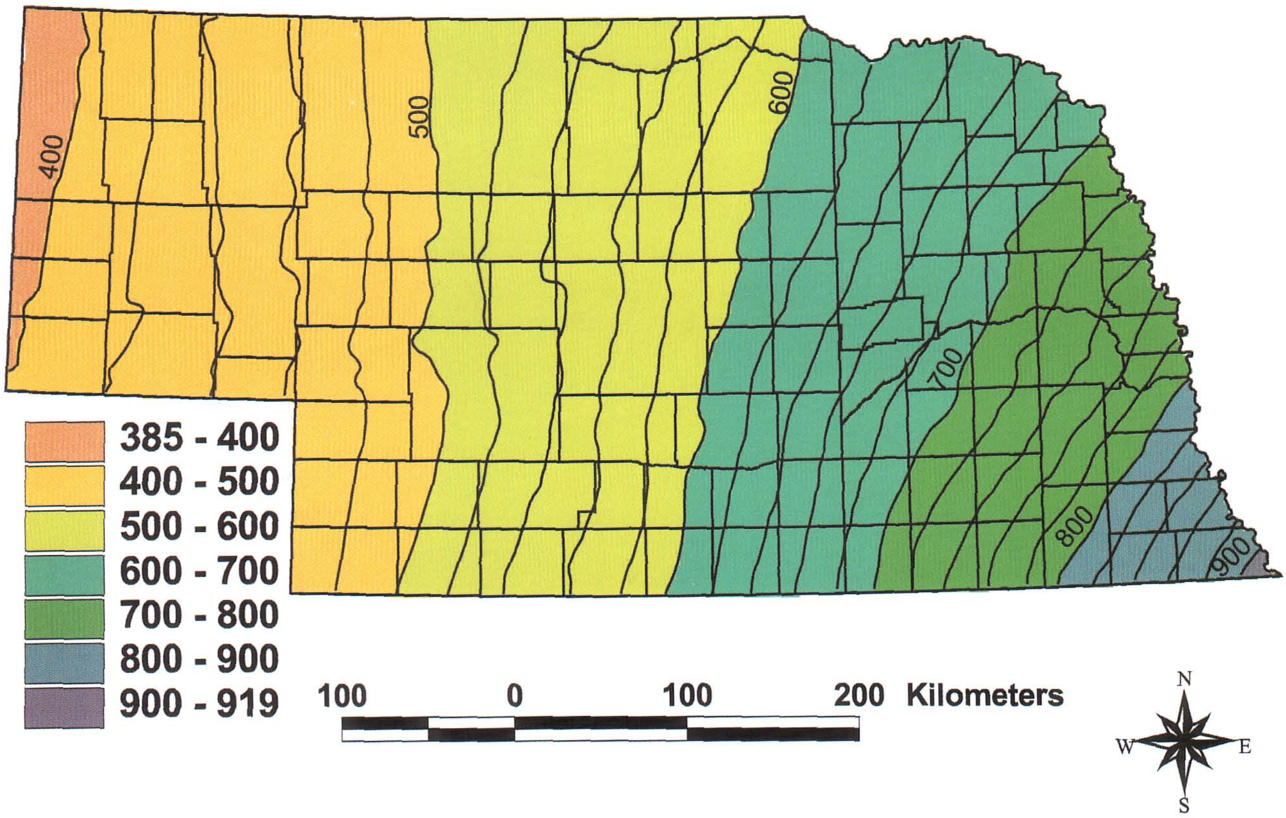

Figure 6. Long-term mean annual precipitation (mm) in Nebraska. The contour interval is 20 mm.

The spatial distribution (Figure 9) of long-term mean annual runoff is obtained by subtracting the *ET* map values from those of the precipitation map, in accordance with Equation 5. Runoff increases by more than 10-fold from

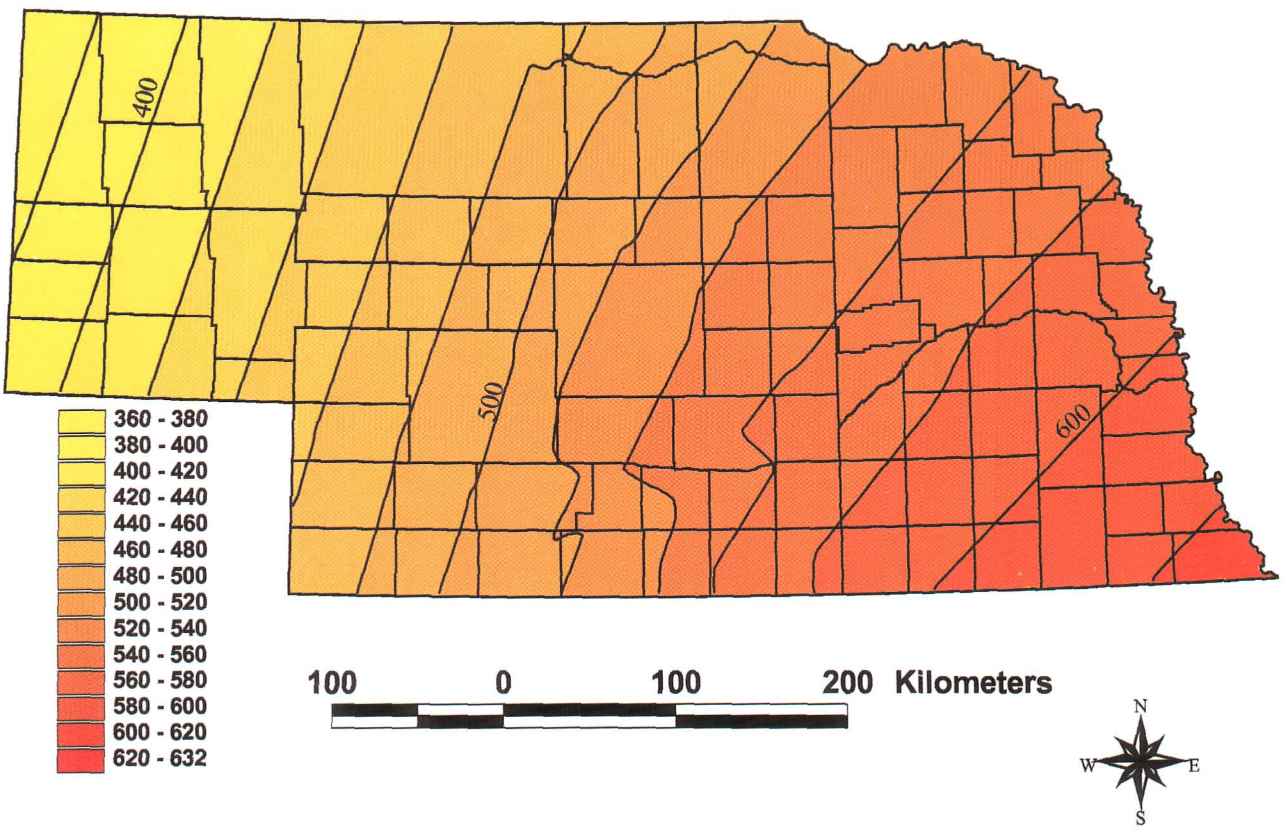

Figure 7. Estimated long-term mean annual evapotranspiration (mm) in Nebraska. The contour interval is 20 mm.

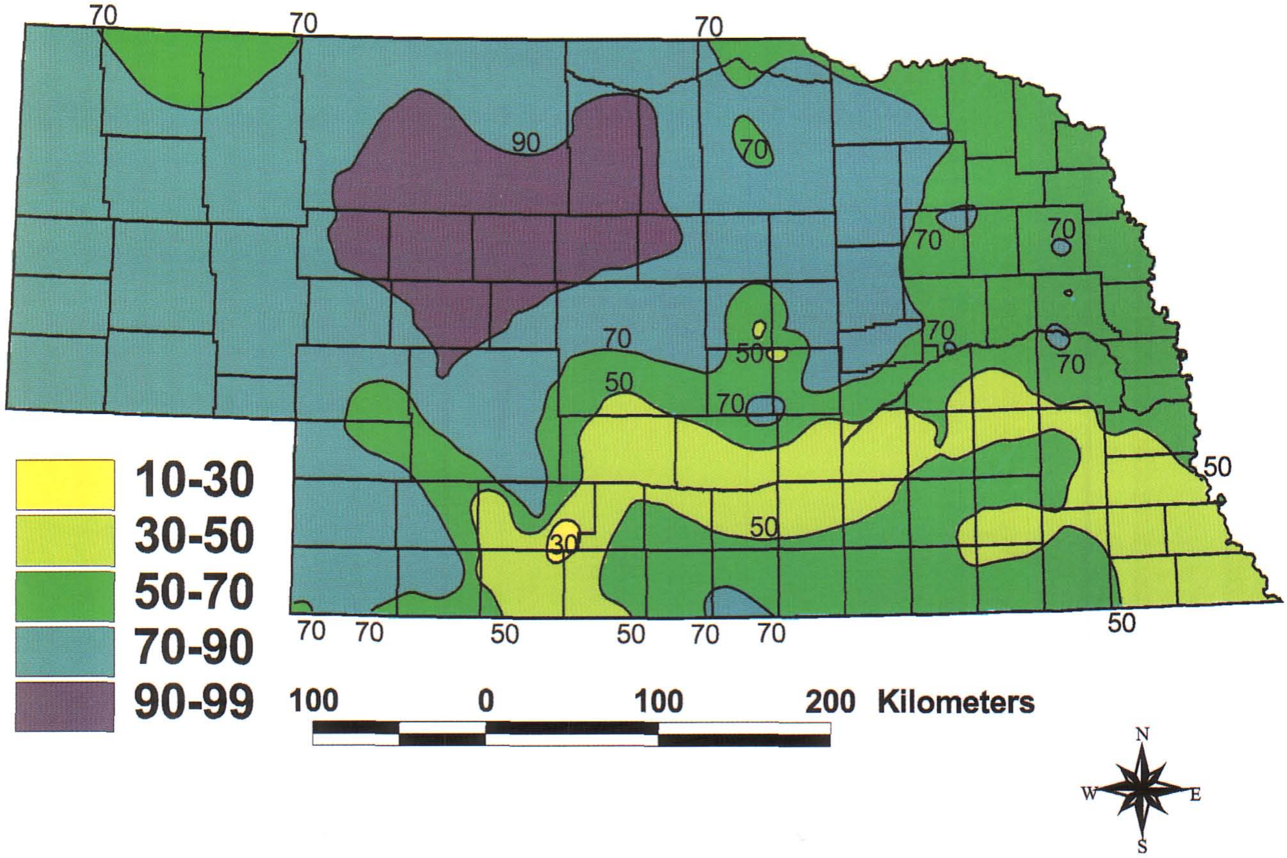

Figure 8. Estimated long-term mean annual base-flow index, BFI (%) in Nebraska.

northwest to southeast and by more than 25-fold from west to east across the Nebraska-Kansas border. This significant difference in runoff across the state is mostly due to the general decrease in annual precipitation and to the increas-

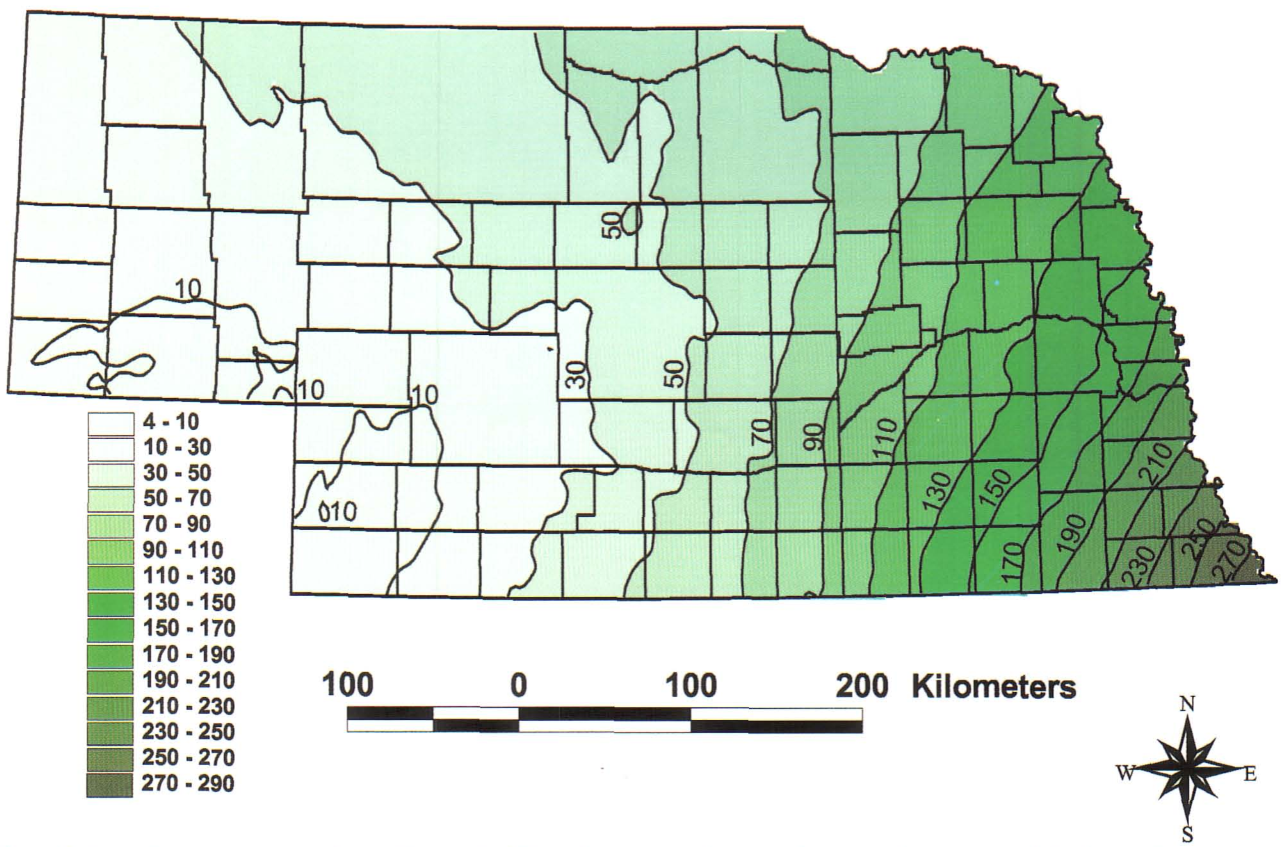

Figure 9. Long-term mean annual runoff (mm) in Nebraska, estimated as the difference between precipitation and evapotranspiration.

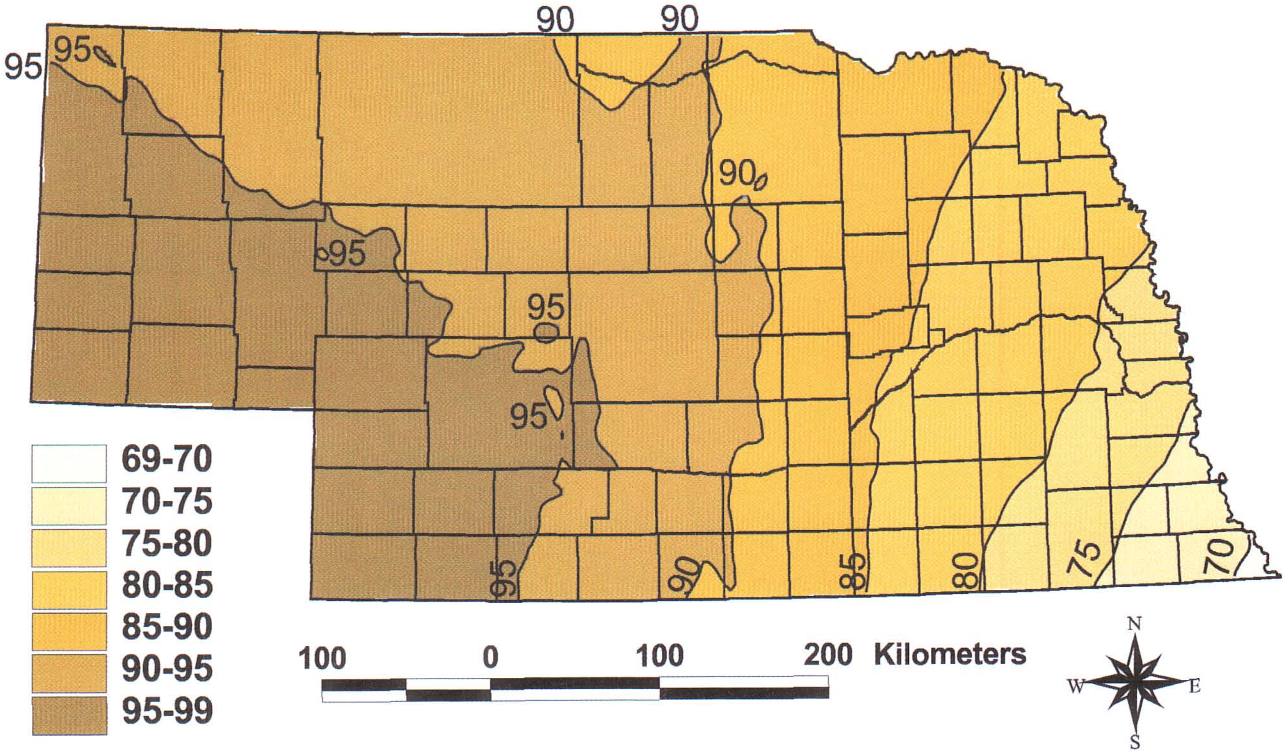

Figure 10. Aridity (%) of the environment in Nebraska. The closer the value to 100%, the more arid the environment becomes.

ing aridity of the environment from east to west. The degree of aridity can be expressed as the ratio of *ET* and precipitation (Figure 10). The closer the value to unity (i.e., 100%), the more arid is the environment. Note the high aridity values in the southwestern and western parts of Nebraska. A long-term mean runoff ratio of 11% for Nebraska can be obtained by dividing the spatial mean (64 mm/year) of the runoff values of Figure 9 by the long-term

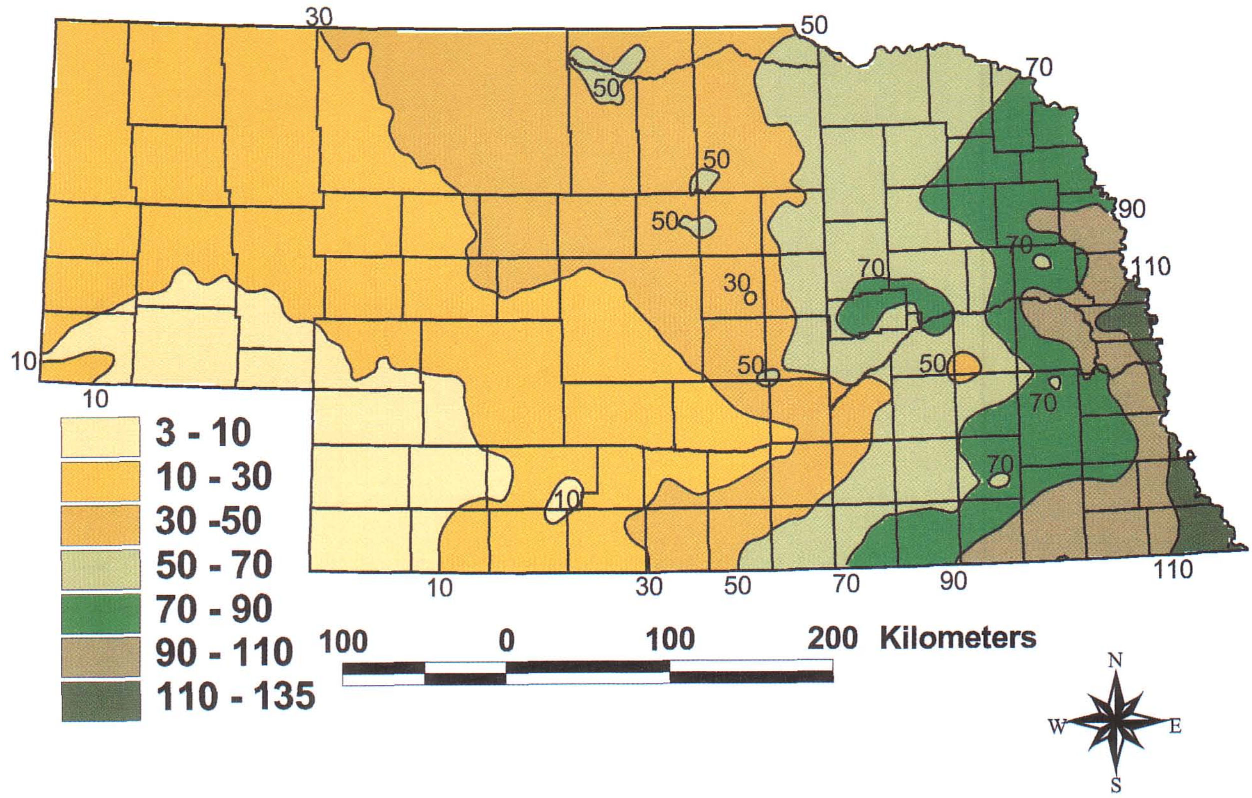

Figure 11. Estimated long-term mean annual base recharge (mm) in Nebraska.

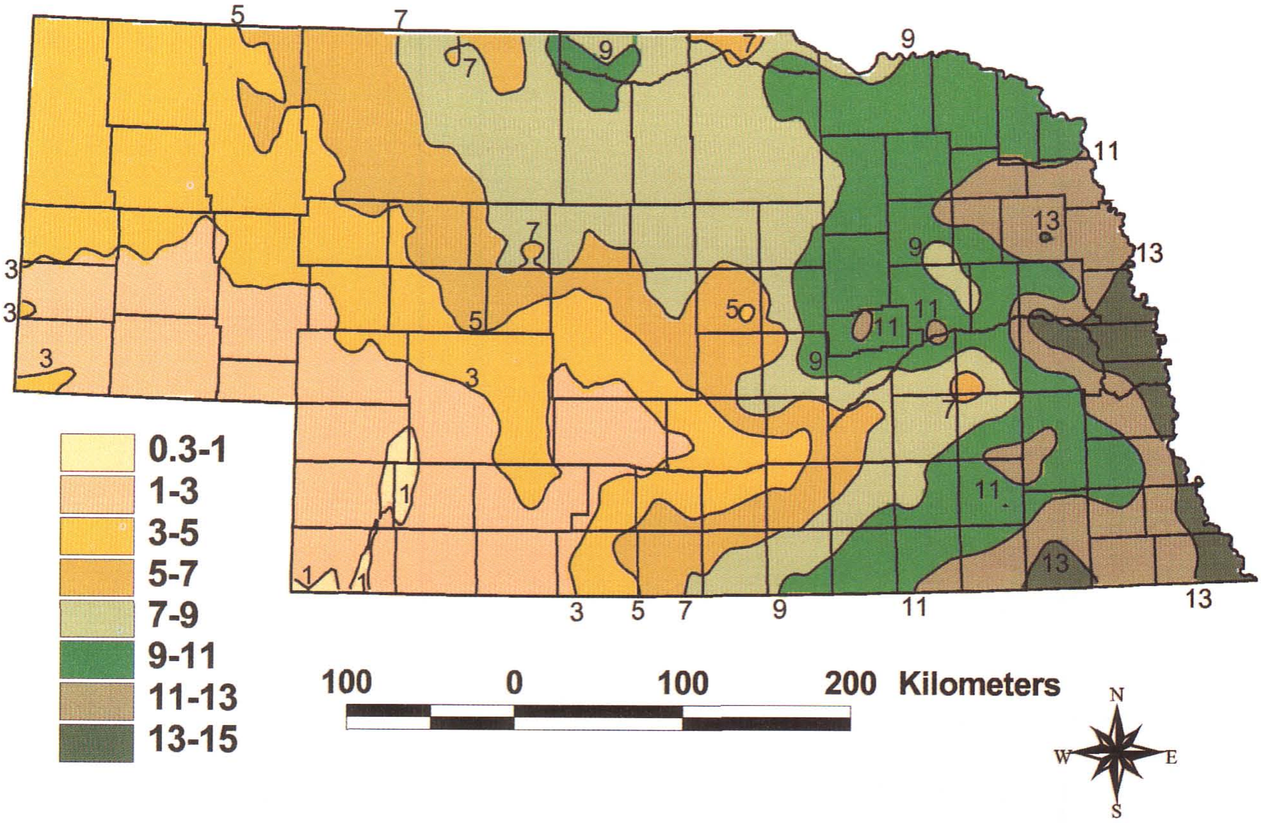

Figure 12. Estimated long-term mean annual base recharge as a percentage of long-term mean annual precipitation in Nebraska.

mean precipitation (575 mm/year, from Figure 6) of the state. It compares favorably with an estimated 10% obtained from detailed long-term water-budget calculations for Nebraska (Dreeszen 2002).

Finally, the spatial distribution of the naturally occurring long-term mean annual base recharge (Figure 11) is obtained by multiplying the runoff map values (Figure 9) with those of the BFI map (Figure 8). The highest rates (>110 mm/year) occur in the easternmost part of Nebraska, primarily due to more abundant precipitation and a less severe aridity index. The central part of the state expresses a rate of ~40 mm annually, with great variability from north to west due to the Sandhills' presence, while western Nebraska receives an annual base recharge of ~15 to 20 mm. These values compare well with long-term mean potential recharge estimates of Dugan and Zelt (2000) obtained by complex hydrologic modeling accounting even for nonirrigated and irrigated crop conditions across the Great Plains. They obtained a long-term mean annual potential recharge rate of ~125 mm/year for eastern Nebraska, 50 mm/year for central Nebraska, and ~15 to 20 mm/year for the western part of the state.

Within the state, the Sandhills region (Figure 3) of north-central Nebraska has long been considered the main recharge area for ground water in the Ogallala Group of the High Plains Aquifer due to the region's highly permeable sandy soils. The high recharge rates are reflected in the high values of the BFI map (Figure 8) and in the increased recharge rates in Figure 11 when compared to areas south of the Sandhills. Because aridity decreases and precipitation increases from the western Sandhills toward its eastern margin, recharge increases as well. Also, recharge increases about twofold (Figures 11 and 12) going from south to north in the middle of the state. Note that in the southwestern part of Nebraska only ~1% of the long-term mean annual precipitation recharges the ground water (Figure 12), while this recharge is larger than 13% of the annual precipitation in the southeasternmost portion of the state. This is mainly because soil moisture in the east is generally closer to field capacity throughout the year due to greater precipitation and a less arid climate.

Summary

Naturally occurring long-term mean annual base recharge at a regional scale can be estimated using a water-balance approach coupled with an automated base-flow separation technique. The water balance uses meteorological and discharge measurements. Geostatistics are used to generate surfaces of variables from point measurements. An objective automated base-flow separation technique is applied to estimate the base-flow index. Areal evapotranspiration estimates were generated by the WREVAP model. Finally, GIS is used to manipulate the maps of the different variables in the water balance.

The techniques used are easy to implement, widely available (such as GIS, geostatistical packages, and the WREVAP model) and do not require complex hydrogeologic modeling nor detailed knowledge of soil characteristics, vegetation cover, or land-use practices. It avoids the use of contributing drainage areas in the water-balance equation, which may be an advantage if it is unknown or when the contributing areas of ground and surface water differ significantly.

The present approach is considered a first step in defining regional-scale recharge, which has not been attempted before on a statewide basis in Nebraska. The technique provides base values against which results of other, more complicated and more data-intensive recharge estimations can be compared. It can also provide input to complex ground water flow models or validate their recharge estimates obtained through parameter optimization.

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Disclaimer: The views, conclusions, and/or opinions expressed in this paper are solely those of the authors and not the University of Nebraska, State of Nebraska or any political subdivision thereof.

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Appendix

Equation 3 can be expressed as

$$Q_b(i) - kQ_b(i-1) = \frac{1-k}{2}[Q(i) + 2Q(i-1) - Q(i-1)] \quad \text{(A1)}$$

which by adding the term $(k\text{–}1)\ Q_b\,(i\text{–}1)$ to both sides of Equation A1 transforms to

$$Q_b(i) - Q_b(i-1) = \frac{1-k}{2}[Q(i) - Q(i-1)] + (1-k)Q(i-1) + (k-1)Q_b(i-1) \quad \text{(A2)}$$

Taking the sum from $i = 1$ to $i = N$, one obtains

$$Q_b(N) - Q_b(0) + (1-k)\sum_{i=1}^{N} Q_b(i-1) = \frac{1-k}{2}[Q(N) - Q(0)] + (1-k)\sum_{i=1}^{N} Q(i-1) \tag{A3}$$

For large N only the summation terms remain, which give

$$BFI = \frac{\sum Q_b}{\sum Q} = 1 \tag{A4}$$

Equation A4 demonstrates that without the imposed constraint of $Q_b(i) \leq Q(i)$ in the filter application, the baseflow index (BFI) would be unity, and that the value of the filter parameter does not impose an artificial limit on the BFI value. This is not the case with the filter recommended by Chapman (1991):

$$Q_b(i) = \frac{3k-1}{3k}Q_b(i-1) + \frac{1-k}{3-k}[Q(i) + Q(i-1)] \tag{A5}$$

which upon expressing the second term in the bracket as $2Q(i-1) - Q(i-1)$ and subtracting $(1/3k)Q_b(i-1)$ from both sides transforms to

$$Q_b(i) - Q_b(i-1) = \frac{1-k}{3-k}[Q(i) - Q(i-1)] - \frac{1}{3k}Q_b(i-1) + \frac{2(1-k)}{3-k}Q(i-1) \tag{A6}$$

Again, taking the sum from $i = 1$ to $i = N$, and neglecting the insignificant terms one obtains

$$BFI = \frac{\sum Q_b}{\sum Q} = 3k\frac{2(1-k)}{3-k} = \frac{6k(1-k)}{3-k} \tag{A7}$$

The right side of Equation A7 has a maximum of ~0.61 at $3-\sqrt{6}$ in the physically interpretable interval of $k < 3$, which demonstrates that BFI cannot exceed 61% with this filter.